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ORTHOGONAL TRANSFORMATIONS FOR WHICH TOPOLOGICAL EQUIVALENCE IMPLIES LINEAR EQUIVALENCE

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Let $R_1, R_2 \in O(n)$, the group of orthogonal transformations of $\mathbf{R}^n$. We say R_1 and R_2 are *topologically* (resp. *linearly*) *equivalent* if there is a homeomorphism (resp. linear automorphism) $f\colon \mathbf{R}^n \longrightarrow \mathbf{R}^n$ such that

$$(1) \qquad f^{-1}R_1 f = R_2\colon \mathbf{R}^n \longrightarrow \mathbf{R}^n, \qquad f(0) = 0.$$

(Of course, linear equivalence of R_1 with R_2 is the same as equality of the respective sets of complex eigenvalues.) The *order* of an orthogonal transformation is its order as an element of $O(n)$. The purpose of this note is to announce and discuss the proof of the following result [HP].

THEOREM A. *Let $R_1, R_2 \in O(n)$ have order $k = l2^m$, where l is odd and $m \geqslant 0$. Suppose that*

(a) *R_1 and R_2 are topologically equivalent, and*

(b) *each eigenvalue of R_1^l and R_2^l is either* 1 *or a primitive* 2^mth *root of unity. Then R_1 and R_2 are linearly equivalent.*

If G is a group and $\rho_1, \rho_2\colon G \longrightarrow O(n)$ are orthogonal representations, we say ρ_1 and ρ_2 are *topologically* (resp. *linearly*) *equivalent* if there is a homeomorphism (resp. linear automorphism) $f\colon \mathbf{R}^n \longrightarrow \mathbf{R}^n$, $f(0) = 0$, such that

$$(2) \qquad f\rho_1(g)(x) = \rho_2(g)f(x),$$

for all $x \in \mathbf{R}^n$, $g \in G$. Here is an equivalent statement of Theorem A giving a more geometric description of its condition (b).

THEOREM B. *Let $\rho_1, \rho_2\colon G \longrightarrow O(n)$ be orthogonal representations of the finite group G such that $\rho_1|H$ and $\rho_2|H$ define semi-free actions of H on $\mathbf{R}^n$ for each cyclic 2-subgroup H of G. If ρ_1 and ρ_2 are topologically equivalent, then they are linearly equivalent.*

Returning to Theorem A, note that if k is odd, condition (b) may be omitted; in this case the result has been proved independently, using rather different methods, by Madsen and Rothenberg [MR]. If k is an odd prime power,

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