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ÉTALE K-THEORY AND ARITHMETIC

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The purpose of this note is to announce some new results about the algebraic K-theory of rings of integers in global fields.

THEOREM 1. *Let $\mathcal{O}$ denote the ring of integers in a number field K (i.e. a finite extension field of the rational numbers* **Q**) *and let l be an odd prime. Then there are natural surjective maps*

$$\text{(1.1)}\quad \mathrm{ch}_{i,k}\colon K_{2i-k}(\mathcal{O}) \otimes \mathbf{Z}_l \longrightarrow H^k(\mathcal{O}[1/l], \mathbf{Z}_l(i)), \qquad k = 1 \text{ or } 2,\ 2i - k > 1.$$

REMARK. The requirement that l be an odd prime can be dropped if K is totally imaginary.

The groups on the right of (1.1) are continuous l-adic étale cohomology groups. Recall that $\mathbf{Z}/l^\nu(1)$ denotes the sheaf of l^νth roots of unity, $\mathbf{Z}/l^\nu(i) = (\mathbf{Z}/l^\nu(1))^{\otimes i}$, and $\mathbf{Z}_l(i) = \varprojlim_\nu \mathbf{Z}/l^\nu(i)$. D. Quillen has conjectured the existence of *isomorphisms* of type (1.1). B. Harris and G. Segal [4] have shown that (1.1) is surjective on torsion if $k = 1$; C. Soulé [6] in many cases proved surjectivity for $k = 2$ with $i < l$.

The surjectivity of (1.1) together with A. Borel's computation of $K_*(\mathcal{O}) \otimes \mathbf{Q}$ [1] gives a new proof of the existence [7] of isomorphisms

$$\text{(1.2)}\qquad \mathrm{ch}_{i,k} \otimes \mathbf{Q}\colon K_{2i-k}(\mathcal{O}) \otimes \mathbf{Q}_l \xrightarrow{\sim} H^k(\mathcal{O}[1/l], \mathbf{Q}_l(i)).$$

In particular, Theorem 1 implies that $\mathrm{ch}_{i,1}$ detects "Borel classes" in $K_{2i-1}(\mathcal{O})$ (i.e. basis elements for $K_{2i-1}(\mathcal{O})$/torsion). This leads to the following corollary, which is consistent with long-standing conjectures about the algebraic K-theory with finite coefficients of the algebraic closure of **Q**.

COROLLARY 2. *For any integers $i \geq 1$ and $\nu > 0$ there exists a finite solvable field extension K' of K with ring of integers $\mathcal{O}'$ such that the image of $K_{2i-1}(\mathcal{O})$/torsion in $K_{2i-1}(\mathcal{O}')$/torsion is divisible by l^ν.*

Conjectures by S. Lichtenbaum [5] and work by Lichtenbaum and others relate the values of the Dedekind zeta function of K at negative integers to the number of elements of finite order in the groups $H^k(\mathcal{O}[1/l], \mathbf{Z}_l(i))$. For example, combining (1) with known properties of Bernoulli numbers gives the new result that $K_{134}(\mathbf{Z})$ contains an element of order 37.

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