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THE EXTENT OF DEFINABLE SCALES

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The notion of a *scale* on a pointset is implicit in the classical proof of the Kondo uniformization theorem for Σ_2^1 sets and in the uniformization theorem of Martin and Solovay [2] for Σ_3^1 sets, from the assumption that measurable cardinals exist. It was formally isolated in Moschovakis [3], where it was shown (granting projective determinacy) that all projective sets admit projective scales – and hence, projective sets can be projectively uniformized. Soon after that it became clear that sets which admit definable scales have many desirable properties in addition to uniformization, and the natural problem arose to determine exactly which sets admit definable scales.

Assuming hyperprojective determinacy, Moschovakis [5] proved that every inductive set admits an inductive scale and asked the obvious next question, whether coinductive sets admit definable scales. This appeared to be a crucial test of the strength of determinacy hypotheses and we included it in the list of Victoria Delfino problems in [1].

We will show here (in outline) that under strong determinacy hypotheses, *every coinductive set indeed admits a definable scale* – which is typically substantially more complicated than the given set. We will also solve completely the general problem of the extent of scales within the model $L(\mathbb{R})$ of sets constructible above the continuum: *if $L(\mathbb{R})$ satisfies the axiom of full determinacy* (AD), *then within $L(\mathbb{R})$, a set admits a scale if and only if it is a Σ_1^2 set* (this settles a conjecture of Solovay). One of the lemmas in the latter proof is a simple and elegant characterization of the Σ_1^2 sets in $L(\mathbb{R})$ which is proved in classical Zermelo-Fraenkel set theory and appears to be an analog of the Shoenfield absoluteness theorem (for Σ_2^1) for this model.

It is convenient to assume the notation and terminology of [6], but we will repeat some of the basic definitions that we need in order to make this note more easily comprehensible. Full proofs and related results will be published elsewhere.

1. Scales on coinductive sets. As usual, a *space* is any product

$$\mathcal{X} = X_1 \times \cdots \times X_n$$

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