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CYCLIC ELEMENTS IN SOME SPACES OF ANALYTIC FUNCTIONS

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DEFINITIONS. 1. A^{-p} $(p > 0)$ is the Banach space of analytic functions $f(z)$ in $U = \{z \in \mathbf{C} \mid |z| < 1\}$ that satisfy $|f(z)| = o[(1 - |z|)^{-p}]$ $(|z| \longrightarrow 1)$ with the norm $\|f\| = \max\{|f(z)|(1 - z)^p\}$ $(z \in U)$. Note that $f_n \longrightarrow f$ in A^{-s} and $g_n \longrightarrow g$ *in* A^{-t} implies $f_n g_n \longrightarrow fg$ in $A^{-(s+t)}$.

2. B^p $(p > 0)$ is the Bergman space, i.e., the "analytic" subspace of $L^p(rdrd\theta)$ in U.

3. $A^{-\infty} = \bigcup A^{-p} = \bigcup B^p$ $(p > 0)$. $A^{-\infty}$ is a linear topological space [1].

4. P is the set of all algebraic polynomials $P(z)$. P is dense in any of the spaces A^{-p}, B^p, $A^{-\infty}$.

5. Let A be any of the spaces A^{-p}, B^p, $A^{-\infty}$ and let $f \in A$. The *ideal generated by f in* A is defined by

$$I(f; A) = \operatorname{clos}\{fP \mid P \in P\}.$$

If f is bounded, then also $I(f; A) = \operatorname{clos}\{fg \mid g \in A\}$.

6. An $f \in A$ is called *cyclic in* A if $I(f; A) = A$.

7. A closed set $E \subset \partial U$ is called a *Carleson set* if its Lebesgue measure $|E| = 0$ and $\Sigma_n |I_n| \log(2\pi/|I_n|) < \infty$, where I_n are the components of $\partial U \setminus E$.

THEOREM. *A singular inner function*

$$s(z) = \exp\{-\frac{\zeta + z}{\zeta - z} dv(\zeta)\},$$

where v is a nonnegative singular measure on ∂U, *is cyclic in any (and hence in all) of the spaces* $A^{-\infty}$, A^{-p}, B^p *if and only if* $v(E) = 0$ *for all Carleson sets E.*

The "only if" part is due to H. S. Shapiro [2]. The case $A^{-\infty}$ was treated in [3]. Some partial results in a different direction are due to Daniel H. Luecking.

Since every A^{-p} is a dense subset of some $B^{p'}$, and vice versa, it suffices to prove the Theorem for A^{-p}. Now we use the following result from [3]; it is, roughly, equivalent to the above Theorem for $A^{-\infty}$.

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