BULLETIN (New Series) OF THE
AMERICAN MATHEMATICAL SOCIETY
Volume 5, Number 3, November 1981

ELLIPTIC OPERATORS AND THE DECOMPOSITION OF TENSOR FIELDS

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CONTENTS

1. Introduction. The decomposition of tensor fields into canonical forms arises as an important step in many problems of mathematics and physics. The classical Helmholtz decomposition (divergence free plus gradient) arises in fluid mechanics and electromagnetism. The Hodge-Kodaira decomposition and its generalizations is an important area of mathematical study. More recently various decompositions of 2-tensors have arisen in the study of general relativity [**6**, **16**, **30**]. There have also been applications in differential geometry [**4**, **15**, **17**] and symplectic structures [**3**].

There are, of course, classical methods for the study of some decompositions. These involve the treatment of a single tensor (or vector) field. This is inadequate for most applications. Often one needs to split entire spaces of tensor fields.

The reason for this is that the desired decomposition usually is a linearization of a nonlinear problem. Let us give a simple example.

The configuration space for the dynamics of an incompressible fluid on a Riemannian manifold (M, g) is the space of volume-preserving diffeomorphisms $\mathcal{D}_\mu$ on M. (Here μ is the canonical volume form determined by the metric g.) For more details see [**21**]. $\mathcal{D}_\mu(M)$ is properly thought of as a constraint space in the set of all diffeomorphisms of M. The Euler or Naviar-Stokes equations yield vector fields on $\mathcal{D}_\mu$. Thus a natural question is whether $\mathcal{D}_\mu$ is a submanifold. The principal tool for studying such a question is the implicit function theorem for Banach spaces.

Received by the editors November 1, 1980.
1980 *Mathematics Subject Classification*. Primary 58G99, 35J15.
[1]This research was supported by National Science Foundation Grant #Phy7901801.

0002-9904/81/0000-0501/$08.00