

## FINITE DIMENSIONAL TEICHMÜLLER SPACES AND GENERALIZATIONS

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### 0. Background.

(A) This paper is an expanded version of one read at the Poincaré Symposium of the American Mathematical Society at Bloomington, Indiana, in April 1980. The subject belongs to the "higher" theory of Riemann surfaces, and some readers may not object to being reminded of the main facts in the "standard" theory.

A *Riemann surface*  $S$  is a connected surface on which one can do complex function theory which is, locally, not distinguishable from ordinary complex function theory in a domain of the complex number plane  $\mathbb{C}$ . More precisely, it is required that  $S$  be a connected Hausdorff space, that certain continuous complex valued functions on subdomains of  $S$  be designated as holomorphic, and that the following propositions be valid. (i) For every point  $P$  of  $S$  there

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