BULLETIN (New Series) OF THE
AMERICAN MATHEMATICAL SOCIETY
Volume 4, Number 2, March 1981

SINGULAR INTEGRALS ON PRODUCT DOMAINS

BY ROBERT FEFFERMAN

Introduction. In their well-known theory of singular integrals on R^n, Calderón and Zygmund [1] obtained the boundedness of certain convolution operators on R^n which generalize the Hilbert transform on R^1. Thus, we know that if $Tf = f * K$ and $K(x)$ is defined on R^n and satisfies the analogous estimates that $1/x$ satisfies on R^1, namely

(i) $|K(x)| \leqslant C/|x|^n$,

(ii) $\int_{\alpha<|x|<\beta} K(x)dx = 0$ for all $0 < \alpha < \beta$,

(iii) $\int_{|x|>2|h|} |K(x + h) - K(x)|dx \leqslant C$ for all $h \neq 0$,

then T is a bounded operator on $L^p(R^n)$ for $1 < p < \infty$. (See Stein [2].)

Now if we take the space $R^n \times R^m$ along with the *two* parameter family of dilations $(x, y) \longrightarrow (\delta_1 x, \delta_2 y)$, $x \in R^n$, $y \in R^m$, $\delta_i > 0$, instead of the usual one parameter dilations, we are led to consider operators which generalize the double Hilbert transform on R^n, $Hf = f * 1/xy$. The boundedness properties of H are usually very easy to obtain by an argument which iterates the one-dimensional theory of the Hilbert transform. But if we consider, more generally, operators $Tf = f * K$ where K satisfies analogous estimates to those satisfied by $1/xy$ but cannot be written in the form $K_1(x) \cdot K_2(y)$ then the argument which deals with H fails.

We wish to announce here that for various classes of kernels K which "look like" $1/xy$ on R^2, but are not products of two functions on the x and y variables respectively, the convolution operators are bounded on L^p for $1 < p < \infty$ and take $L \log^+ L(R^n \times R^m)$ boundedly to weak L^1. In particular this involves the problem of formulating the right two parameter versions of the assumptions on the kernel K.

We wish to take this opportunity to thank E. M. Stein for his help in the course of this work. The formulation of several of our theorems follows his suggestions, and Theorem 3 in its L^p form is due to him. We also wish to thank the Institute for Advanced Study for its hospitality and the National Science Foundation for its financial support.

Statement of results. We shall state three results dealing with the action of convolution operators. These deal with the action of these operators on L^2, L^p for $1 < p < \infty$, and $L \log^+ L$ respectively.

Received by the editors June 18, 1980.

1980 *Mathematics Subject Classification*. Primary 42A40.