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Dedicated to my teacher Flavio Previale

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1. Introduction. The notion of algorithm permeates the experience of every mathematician, and goes back to the very origins of mathematics. To answer positively a problem asking for an algorithmic solution, the intuitive notion of algorithm is sufficient since we can simply exhibit a solution and convince ourselves that the procedure is effective. This has been done for centuries. If we have instead reasons to believe that the answer is negative, we need a formal characterization of algorithm to prove that each one is ruled out as a solution. Such a characterization was only obtained in the late 1930's with the work of Herbrand, Gödel, Church, Turing, Post and Kleene. (See **[Da1]**.) They defined various notions of recursive function (on the set of natural numbers) and the related concept of recursively enumerable (r.e.) set, using for example abstract machines, routine rules of calculation or generative grammars. All of these prima facie different approaches defined the same class of functions and of sets. These are the precise analogues of the intuitive notions of computable function and effectively generated set.

With the formal version of computability at hand, we can talk of an

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