15. Ju. G. Reshetnyak, *On the stability of conformal mappings in multi-dimensional spaces*, Sib. Mat. Zhurnal **8** (1967), 91–114.(Russian)

16. O. Teichmüller, *Extremale quasikonforme Abbildungen und quadratische Differentiale*, Preuss. Akad. Wiss. **22** (1940), 1–197.

F. W. GEHRING

BULLETIN (New Series) OF THE
AMERICAN MATHEMATICAL SOCIETY
Volume 3, Number 3, November 1980

0002-9904/80/0000-0515/$02.00

Optimization and approximation, by W. Krabs, translated by Philip M. Anselone and Ronald B. Guenther, John Wiley and Sons, Chichester-New York-Brisbane-Toronto, 1979, xii + 220 pp., $28.95.

0. Introduction. Although approximation theory is a discipline with very wide spectrum overlapping with areas ranging from classical function theory to operator theory, the very basic problems are those that deal with best approximation. More precisely, questions of existence, uniqueness, and characterization of the best approximants from a certain subspace or a certain submanifold, as well as the questions of determination and estimation of the orders of best approximation are of utmost importance. Very frequently, with the possible exception of the problem of uniqueness, the techniques and theory of optimization turn out to be quite useful. This book is devoted to the connection between best approximation and optimization; it contains numerous examples showing how approximation problems of different types arising in physical as well as technical applications can be treated in a unified way within the general framework of optimization theory. The main body of the book consists of three chapters: Chapter I treats the general linear theory of optimization including the standard existence and duality theorems, while Chapters II and III deal with the convex problems and the general nonlinear problems respectively. Each of these chapters begins with a series of examples followed by the development of the theoretical principles. The advantage of this presentation is that each chapter can be read almost independently. An appendix on the background of functional analysis is also included. We will divide our discussion into three parts, each of which deals with one chapter of the book.

I. Linear problems. The general topic treated throughout this book is that of continuous approximation problems. These lead to semi-infinite (completely infinite) optimization problems in which finitely (infinitely) many free parameters can occur along with infinitely many side conditions. Such infinite optimization problems arise when one seeks to approximate a continuous real-valued function f on a compact set S by linear combinations of continuous functions on S as well as possible in the sense of the maximum norm. As the author mentions, such a situation occurs when one seeks simplified representations of functions for the purpose of evaluation on a computer. This situation also occurs in the approximate solution of boundary and initial value problems for ordinary and partial differential equations.

There are many such examples scattered throughout the book. A typical problem, as given by Krabs, concerns mathematically treating the problem of