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BOOK REVIEWS

Quasiconformal mappings and Riemann surfaces, by Samuil L. Krušhkal′, edited by Irwin Kra, John Wiley and Sons, New York, 1979, xii + 319 pp., $24.95.

Under review is a book on quasiconformal mappings and their role in the study of Riemann surfaces and Kleinian groups. Since quasiconformal mappings are beginning to find applications outside of complex analysis (see, for example, [5]), it may be worthwhile to begin by listing some basic facts about them.

Suppose that D and D' are domains in euclidean n-space R^n, $n \geq 2$, and suppose that f is a sense preserving homeomorphism of D onto D'. For each x in D set

$$H(x, f) = \limsup_{r \to 0} \frac{\max_{|y-x|=r} |f(y) - f(x)|}{\min_{|y-x|=r} |f(y) - f(x)|}.$$

If $H(x, f)$ is bounded in D, then f is said to be *quasiconformal* and

$$K(f) = \operatorname*{ess\,sup}_{x \in D} H(x, f)$$

is called the *maximal dilatation* of f. Obviously $1 \leq K(f) < \infty$. When $n = 2$, $K(f) = 1$ if and only if f, regarded as a complex valued function of a complex variable, is analytic in D [13]. When $n \geq 3$, $K(f) = 1$ if and only if there exists a Möbius transformation, i.e. the composition of an even number of reflections in $(n - 1)$-spheres and planes, which coincides with f in D, [7] and [15]. Thus f is *conformal* if and only if it is quasiconformal with $K(f) = 1$.

Compositions and inverses of quasiconformal mappings are again quasiconformal and

$$K(f \circ g) \leq K(f)K(g), \qquad K(f^{-1}) = K(f).$$

Thus the quasiconformality and maximal dilatation of a given mapping f remain unchanged after composition with conformal mappings, and one can extend these concepts to homeomorphisms between two *conformally flat n-dimensional manifolds* S and S', i.e. n-dimensional manifolds for which the transition mappings are conformal. Of central importance is the case $n = 2$ when S and S' are Riemann surfaces.

Plane quasiconformal mappings have been studied for over fifty years. They occur first in the papers of Grötzsch, who posed and solved several important and suggestive extremal problems in [9]. They appear next in Ahlfors' fundamental paper on covering surfaces [1]; here one first sees that the geometric aspects of value distribution theory hold for functions which are locally uniformly quasiconformal rather than just locally conformal. Forty years later Drasin appealed to this fact to obtain his complete solution