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HOMOLOGY OF GROUP SYSTEMS WITH APPLICATIONS TO LOW-DIMENSIONAL TOPOLOGY

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Eilenberg-Mac Lane complexes are generalized to *GEM complexes.* This generalization is then shown to unify many diverse seemingly unrelated concepts in low-dimensional topology. All 2-dimensional *CW*-complexes [1], all 3-dimensional manifolds [5], and all smooth 2-knot exteriors [5] are shown to be GEM complexes. A method is given for computing the (co)homology of the universal cover of a GEM complex from the (co)homology of a naturally associated group system. Hence, this yields a method for computing the second homotopy group π_2 and the k-invariant in $H^3(\pi_1; \pi_2)$.

I. GEM complexes.

DEFINITION. A *generalized Eilenberg-Mac Lane* (*GEM*) *complex* is a *CW*-complex K together with nonempty subcomplexes K_-, K_0, K_+ such that (1) $K = K_- \cup K_+$, (2) $K_0 = K_- \cap K_+$, (3) each K_λ is 0-connected and aspherical (i.e., $\pi_q K_\lambda = 0$ for $q \neq 1$) for $\lambda = -, 0, +$. The *associated group system* $\mathbf{G} = \pi_1 \mathbf{K}$ is the collection of groups, $\{\pi_1 K_-, \pi_1 K_0, \pi_1 K_+\}$ together with the morphisms induced by inclusion.

THEOREM 1. *Let K and K' be two GEM complexes. If an associated group system $\pi_1\mathbf{K}$ of K is isomorphic to an associated group system $\pi_1\mathbf{K}'$ of K', then K and K' are of the same homotopy type. Hence, the name "GEM" and the notation $K = K(\mathbf{G}, 1)$ are justified.*

THEOREM 2. *For every group system* $\mathbf{G}$, *the GEM complex* $K(\mathbf{G}, 1)$ *exists.*

REMARK 1. The exterior of every smooth 2-knot (S^4, kS^2) is a GEM complex since every 2-knot has a hyperbolic splitting. (See [5].) (This is a natural 4-dimensional analogue of the asphericity of classical knots [9].) Every 3-manifold is a GEM complex since every such 3-manifold has a Heegaard splitting of positive genus. Every 2-dimensional *CW*-complex has a subdivision which is a GEM complex [1].

II. Group systems.

DEFINITION. Let $\mathbf{G}$ be a group system and let G denote its direct limit

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