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BLOCKS WITH CYCLIC DEFECT GROUPS IN $GL(n, q)$

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Let G be a finite group and B an r-block of G with cyclic defect group R. The decomposition of the ordinary characters in B into modular characters is described by the Brauer tree T of B. The problem of determining the Brauer trees for finite groups of Chevalley type was proposed by Feit at the 1979 AMS Summer Institute. Our result is a necessary step in this problem: If $G = GL(n, q)$ and r is an odd prime not dividing q, then T is an open polygon with its exceptional vertex at one end. The proof also shows an interesting fit of the modular theory for such primes r with the underlying algebraic group, the Deligne-Lusztig theory, and Young diagrams.

Because R is a cyclic defect group, R has the form

$$R = \begin{pmatrix} I_l & 0 \\ 0 & R_1 \end{pmatrix}, \tag{1}$$

where the elementary divisors of a generator of R_1 are, say, m copies of an irreducible polynomial of degree d over F_q. By (1) the structure of $C = C_G(R)$ is

$$C = \begin{pmatrix} C_0 & 0 \\ 0 & C_1 \end{pmatrix}, \tag{2}$$

where $C_0 \simeq GL(l, q)$ and $C_1 \simeq GL(m, q^d)$. The normalizer $N = N_G(R)$ is then obtained by adjoining to C an element t of the form

$$t = \begin{pmatrix} I_l & 0 \\ 0 & t_1 \end{pmatrix},$$

where t_1 induces a field automorphism of order d on C_1.

By Brauer's First Main Theorem B corresponds to a block B_C of C with defect group R, where B_C is determined up to conjugacy in N. Let E be the stabilizer of B_C in N, so $e = |E: C|$ is then the inertial index of B. Let Λ be a set of representatives for the orbits of E on the set of nontrivial irreducible characters of R. In the Brauer-Dade theory [1] the exceptional characters χ_λ in B are labeled by λ in Λ, the nonexceptional characters χ_i in B by $i = 1, 2, \ldots, e$, and the $e + 1$ vertices of T by $\chi_1, \chi_2, \ldots, \chi_e$, exc.

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