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ABSENCE OF SINGULAR CONTINUOUS SPECTRUM IN N-BODY QUANTUM SYSTEMS[1]

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ABSTRACT. For a large class of potentials including arbitrary bounded potentials with $r^{-2-\epsilon}$ falloff and also allowing suitable local singularities and slower falloff, we demonstrate that the singular continuous spectrum of N-body quantum Hamiltonians is empty. We accomplish this by extending Mourre's work on three body problems to N-bodies.

We want to consider here multiparticle Schrödinger operators, i.e. the Hamiltonian operators of N-body quantum systems. Given a function, V_γ, on R^ν for each pair $\gamma \subset \{1, \ldots, N\}$, the operator

$$\widetilde{H} = -\sum_{i=1}^{N} (2m_i)^{-1} \Delta_i + \sum_{\gamma} V_\gamma(r_\gamma) \tag{1}$$

on $L^2(R^{N\nu})$ is the Hamiltonian before removal of the center of mass. In (1), we write $r \in R^{\nu N}$ as $(r_1, \ldots, r_N)$; let Δ_i be the Laplacian with respect to r_i and if $\gamma = (ij)$, we write $r_\gamma = r_i - r_j$. If one decomposes $L^2(R^{N\nu}) = \mathcal{H} \otimes \mathcal{H}_{cm}$ with the first factor functions of r_γ and the second functions of $R = (\Sigma m_i)^{-1}(\Sigma m_i r_i)$, then $\widetilde{H} = H \otimes 1 + 1 \otimes T_{cm}$ with $T_{cm} = (2\Sigma m_i)^{-1} \Delta_R$ (see, e.g. [10]). H is the Schrödinger operator we want to discuss. There are three main features of the spectrum of H which one wants to establish in cases where V_γ has suitable falloff at $r_\gamma \longrightarrow \infty$.

(i) Point spectrum can only accumulate at thresholds.

(ii) H has no singular continuous spectrum.

(iii) Scattering is complete.

Thresholds are defined as follows: Let a be a partition of $\{1, \ldots, N\}$ and write $\gamma \subset a$ if γ is a subset of one of the clusters in a. Write $H = H(a) + I(a)$ with $I(a) = \Sigma_{\gamma \not\subset a} V_\gamma$ and write $\mathcal{H} = \mathcal{H}^a \otimes \mathcal{H}_a$ with the first factor functions of r_γ with $\gamma \subset a$ and the second functions of differences of centers of mass of distinct clusters in a. Then $H(a) = H^a \otimes I + I \otimes T_a$: H^a is the Hamiltonian of the internal motion of the clusters and T_a the kinetic energy of motion of the

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