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TILINGS WITH CONGRUENT TILES

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Introduction. The purpose of this paper is to survey recent results related to the second part of Hilbert's eighteenth problem (see Hilbert [1900]). This problem, which is concerned with tilings of Euclidean space by congruent polyhedra, will be stated below after the necessary terminology has been introduced. Although Hilbert's original question was answered by one of his assistants (Karl Reinhardt) more than fifty years ago, there remain many unsolved problems in this area of mathematics; a description of recent results therefore seems to us to be of considerable interest.

Three surveys of developments related to Hilbert's problems (Aleksandrov [1969], Browder [1976], Kaplansky [1977]) have been published in recent years, but they contain no mention of the remarkable advances made during the last decades in connection with the problem that concerns us. An even more cogent reason for publishing this survey is that much of the material results from the work of crystallographers, and no mention of it appears in the recognized mathematical literature. Reasons for this disregard are easy to find. Current fashions in mathematics applaud abstraction for its own sake, regarding it as the highest intellectual activity–whether or not it is, in any sense, useful or related to other endeavors. Mathematicians frequently regard it as demeaning to work on problems related to "elementary geometry" in Euclidean space of two or three dimensions. In fact, we believe that many are unable, both by inclination and training, to make meaningful contributions to this more "concrete" type of mathematics; yet it is precisely these and similar considerations that include the results and techniques needed by workers in other disciplines. Moreover, throughout the history of mathematics, fundamental ideas for other branches of mathematics have been motivated by "intuitive" geometric questions; irrational numbers, calculus, axiomatics and topology are only a few of the most obvious "inventions" that originated in this manner. It seems to us to be foolish and presumptuous to believe that ours is the first generation which needs no more the inspiration that can be found in studying simple geometric objects and their mutual relations. By showing that there are still many open and difficult (yet interesting and easily understood) problems, we hope to persuade some readers that this is an area of mathematics worthy of their attention.

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