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The theory of Lie superalgebras; an introduction, by M. Scheunert, Lecture Notes in Math., vol. 716, Springer-Verlag, Berlin-Heidelberg-New York, vi + 271 pp.

A Lie superalgebra, or $(\mathbb{Z}_2\text{-})$ graded Lie algebra, is a vector space $\mathfrak{G} = \mathfrak{G}_0 \oplus \mathfrak{G}_1$ with a bilinear multiplication, \langle , \rangle , satisfying the graded versions of the axioms for Lie algebras: if $X \in \mathfrak{G}_\alpha$, $Y \in \mathfrak{G}_\beta$, and $Z \in \mathfrak{G}_\gamma$ ($\alpha, \beta, \gamma \in \{0, 1\}$), then

$$(1) \langle X, Y \rangle = (-1)^{\alpha\beta} \langle Y, X \rangle \text{ (“graded antisymmetry”);}$$

$$(2) (-1)^{\alpha\gamma} \langle X, \langle Y, Z \rangle \rangle + (-1)^{\beta\alpha} \langle Y, \langle Z, X \rangle \rangle + (-1)^{\gamma\beta} \langle Z, \langle X, Y \rangle \rangle = 0$$

(the “graded Jacobi identity”).

Note that \mathfrak{G}_0 is a Lie algebra (in the ordinary sense). In what follows, it will always be tacitly assumed that \mathfrak{G} is finite dimensional and is defined over a field of characteristic 0.

The standard example of an ordinary Lie algebra is $gl(n)$, the space of all $n \times n$ matrices, with $[X, Y] = XY - YX$. (For instance, a representation of a Lie algebra is a homomorphism into $gl(n)$.) There is a corresponding standard example of a Lie superalgebra; it, too, is used to define representations. Let $V = V_0 \oplus V_1$ be a \mathbb{Z}_2 -graded vector space. We define $pl(V) = pl(V)_0 \oplus pl(V)_1$, where

$$pl(V)_0 = \{ V \rightarrow V, T(V_j) \subseteq V_j, j = 0, 1 \};$$

$$pl(V)_1 = \{ S: V \rightarrow V: S(V_j) \subseteq V_{1-j}, j = 0, 1 \};$$

thus $pl(V)_0$ consists of the linear maps on V taking each distinguished subspace to itself, and $pl(V)_1$ consists of the linear maps on V taking each to the other. The multiplication is given as follows: if X, Y are each in $pl(V)_0$ or $pl(V)_1$, where

$$\langle X, Y \rangle = XY - YX \text{ if either } X \text{ or } Y \in pl(V)_0;$$

$$\langle X, Y \rangle = XY + YX \text{ if } X, Y \in pl(V)_1.$$

Thus the multiplication in $pl(V)$ consists of both commutators and anticommutators. It is this fact which explains the sudden interest in Lie superalgebras among physicists; they offer a mathematical framework for combining various symmetry theories. (It seems to be somewhere between unclear and dubious, however, whether the resulting supersymmetry theories do jibe with