[P] T. Przymusiński, *A Lindelöf space X such that $X^2$ is normal but not paracompact*, Fund. Math. **78** (1973), 291–296.

[$R_1$] M. E. Rudin, *Interval topology in subsets of totally orderable spaces*, Trans. Amer. Math. Soc. **118** (1965), 376–389.

[$R_2$] ______, *Souslin's conjecture*, Amer. Math. Monthly **76** (1969), 1113–1119.

[S] R. Sorgenfrey, *On the topological product of paracompact spaces*, Bull. Amer. Math. Soc. **53** (1947), 631–632.

[V] N. V. Veličko, *Ordered p-spaces*, Izv. Vysš. Učebn Zaved. Matematika **1976**, no. 9 (172), 25–36.

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*Equations of evolution*, by Hiroki Tanabe, translated from Japanese by N. Mugibayashi and H. Haneda, Monographs and Studies in Mathematics, No. 6, Pitman, London-San Francisco-Melbourne, 1979, xii + 260 pp., $42.00.

Many mixed problems i.e. initial value-boundary value problems for partial differential equations can be written in the form

$$du(t)/dt = A(u(t)), u(0) = f. \tag{1}$$

Here the unknown function $u$ maps nonnegative time $t \in \mathbf{R}^+ = [0, \infty)$ into a Banach space $X$, $A$ is an operator acting on its domain $\mathfrak{D}(A) \subset X$ to $X$, and the initial data $f$ is in $\mathfrak{D}(A)$. The boundary conditions are absorbed into the description of $\mathfrak{D}(A)$, and saying that the solution takes values in $\mathfrak{D}(A)$ amounts to saying that the (time independent) boundary conditions hold for all $t$. We assume that $A$ is a densely defined linear operator, and we are interested in the case when the problem (1) is well posed, i.e. a solution exists, it is unique, and it depends continuously (in a suitable sense) on the ingredients of the problem, viz. $f$ and $A$. When this is the case let $T(t)$ map the solution at time 0 (i.e. $f$) to the solution at time $t$ (i.e. $u(t)$). Then the uniqueness gives the semigroup property $T(t)T(s) = T(t + s)$ for $t, s \in \mathbf{R}^+$, and we have $T(t) =$ "$e^{tA}$" at least formally; but in general $A$ is an unbounded operator so one must be careful.

The Hille-Yosida-Phillips theory of (one parameter strongly continuous) semigroups of (linear) operators makes this all precise. The theory says that (1) is well posed iff it is governed by a semigroup $T = \{T(t): t \in \mathbf{R}^+\}$ iff $A$ generates a semigroup $T$; and moreover, $A$ generates a semigroup $T$ iff $A$ satisfies certain explicitly verifiable conditions. For instance, when the semigroup is contractive i.e. $\|T(t)\| \leq 1$ for all $t \geq 0$, the exponential formula

$$T(t)f = \lim_{n\to\infty} \left(I - \frac{t}{n}A\right)^{-n} f$$

suggests that $T$ can be recovered from $A$ if $(I - \lambda A)^{-1}$ is an everywhere defined contraction (i.e. $\|(I - \lambda A)^{-1}\| \leq 1$) for each $\lambda > 0$. In this case $A$ is called $m$-dissipative, and this condition is both necessary and sufficient for $A$ to generate a contraction semigroup.