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SPHERICAL FIBRATIONS

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ABSTRACT. In [8], [9] we extend some theorems of I. M. James and J. H. C. Whitehead on the homotopy type of spherical fibrations. Here we sketch our results and methods.

1. Introduction. Let E_1, E_2 be q-sphere Hurewicz fiberings ($q \geqslant 1$) over the same connected finite CW-complex B. We suppose that the spaces E_1, E_2 have the same homotopy type ($E_1 \simeq E_2$), that E_2 has a cross-section, and that E_2 is orientable. Then by [9, Theorem 1] E_1 is also orientable. We suppose that B is *nilpotent* [1], and let $E_{j(p)} \longrightarrow B_{(p)}$ be the localization of $E_j \longrightarrow B$ at a prime p ($j = 1, 2$).

THEOREM 1. *If* $\dim B < 2q$ *then, for any prime* p, $E_{1(p)}$ *has a cross-section.*

In [3] I. M. James and J. H. C. Whitehead prove a similar result for the case where B is a sphere, but where the fibre of E_1, E_2 is not necessarily a sphere. We comment on our proof in §2.

THEOREM 2. *If* $E_1 \simeq B \times S^q$ *where* E_1 *has a cross-section then* E_1 *is fibre homotopy trivial.*

In [4] James and Whitehead take B to be a sphere and prove a similar result. Our proof in [9] uses a counting argument in the spirit of [3], [7]. Comparing this proof with Theorem 1 we find that if $E_1 \simeq B \times S^q$ where $\dim B < 2q$ then E_1 is fibre homotopy trivial. It was a conjecture along these lines by I. M. James that led me to write [8], [9]. I wish to thank Professor James for telling me his conjecture.

Recall that the *fibre suspension* [6] $\Sigma E \longrightarrow B$ of a map $\pi : E \longrightarrow B$ is defined as follows. Let ΣE be the quotient of $E \times [-1, 1]$ by the relations $(e, -1) \sim (e', -1)$ and $(e, 1) \sim (e', 1)$ for $\pi(e) = \pi(e')$. Then the projection of ΣE takes $[e, t]$ to $\pi(e)$. We assume that E_2 has the fibre homotopy type of ΣE for some E.

REMARKS. (i) If $\dim B < q$ then this assumption holds automatically.

(ii) If E_2 is a fibre bundle then this assumption is equivalent to the requirement that E_2 have a cross-section.

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