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RIEMANN-ROCH THEOREMS FOR HIGHER ALGEBRAIC K-THEORY

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In [1] and [2] Baum, Fulton and MacPherson, generalizing the celebrated Grothendieck-Riemann-Roch theorem, proved that given a category $\mathcal{V}$ of quasi-projective schemes there is a natural transformation called the Todd class of functors (covariant for proper morphisms) between K_0', the homology algebraic K-theory of coherent sheaves and any of the standard homology theories. Here we announce generalizations of the results of [1] and [2] to Quillen's higher algebraic K-theory [8] which may help to illuminate the relationship between algebraic K-theory and more ordinary cohomology theories.

The statements of our theorems depend on defining global analogues of Quillen's construction of Chern classes for the K-theory of a ring [3], [9]. We can use any of the standard cohomology theories defined on $\mathcal{V}$, such as étale or crystalline cohomology or even the Chow ring. All of these theories can be realized for each $X \in \mathcal{V}$ as the hypercohomology of a graded complex or pro-complex Γ_j^*, $j \in \mathbf{Z}$, of sheaves on the Zariski site of X. All of these theories have Chern classes for representations of sheaves of groups and there exist universal classes

$$C_i \in \mathbf{H}^{di}(X, GL(\mathcal{O}_X), \Gamma_i^*) \qquad (d = 1 \text{ or } 2).$$

Using Brown's generalized cohomology "with supports" of simplicial sheaves [6], and the functor $\mathbf{Z}_\infty$ of [5] instead of the "+" construction one can mimic in the category of simplicial sheaves the methods of [3] and [9] to obtain Chern classes for all $p > 0$

$$C_{i,p}^Y; K_p^Y(X) = K_p(X, X - Y) \longrightarrow \mathbf{H}_Y^{di-p}(X, \Gamma_i^*)$$

whose domains are the relative K-groups, defined so as to force a Quillen-style localization sequence. One can show that these classes coincide for $p = 0$ with those of Iversen [7]. For $p > 0$ they are group homomorphisms and are compatible with products in the way described by Bloch [3], hence one can define a Chern character with supports, which is a ring homomorphism

$$ch^Y\colon \bigoplus_{p \geq 0} K_p(X, X - Y) \longrightarrow \bigoplus_{i,p \geq 0} \mathbf{H}_Y^{di-p}(X, \Gamma_i^*) \otimes \mathbf{Q}.$$

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