ON THE ROLE OF THE HEISENBERG GROUP
IN HARMONIC ANALYSIS

BY ROGER HOWE

In this article I want to popularize the Heisenberg group, which is remarkably little known considering its ubiquity. I use the word ubiquity advisedly. To justify it, let me give a sample of the many apparently diverse topics where the Heisenberg group reveals itself as an important factor.

(1) Representation Theory of Nilpotent Lie Groups

(2) Foundations of Abelian Harmonic Analysis

(3) Moduli of Abelian Varieties

(4) Structure Theory of Finite Groups

(5) Theory of Partial Differential Equations

(6) Quantum Mechanics

(7) Homological Algebra

(8) Ergodic Theory

(9) Representation Theory of Reductive Algebraic groups

(10) Classical Invariant Theory

This list could easily be lengthened both by adding new topics and making these more specific, for sometimes the applications are multiple. In fact, one of the most important areas of application I have not mentioned at all, to avoid name-dropping. Why has an object with such wide application gone relatively unnoticed until recently? One can only speculate. One reason might be that the role of the Heisenberg group in many situations is relatively subtle. An investigator might be able to get what he wanted out of a situation while overlooking the extra structure imposed by the Heisenberg group, structure which might enable him to get much more. Such may have been the case with Hermann Weyl, one of the pioneers in introducing the Heisenberg group into Quantum Mechanics [Wy1]. Indeed, many physicists still call the Heisenberg group the "Weyl group". When Weyl wrote his book *The classical groups*, [Wy2] he overlooked the natural occurrence of the Heisenberg group, exploitation of which yields results which one feels Weyl would have liked very much. Again, I gather that an appreciation of the role of the Heisenberg group in rigidifying abelian varieties was an important aspect of Mumford's [Mm] fundamental contributions to their study. Another obstacle to the appreciation of the common underlying structure may have been the very diversity of the topics I listed above, for detection of its presence in one place need not suggest its presence elsewhere. Indeed, investigators in one field may very well never have been aware that the Heisenberg group had been found in some field not seemingly related to theirs. Another factor certainly contrib-