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Generalized functions: Volume 1, *Properties and operations*, xviii + 423 pp.; Volume 2, *Spaces of fundamental and generalized functions*, x + 261 pp.; Volume 3, *Theory of differential equations*, x + 222 pp.; Volume 4, *Applications of harmonic analysis*, xiv + 384 pp.; Volume 5, *Integral geometry and representation theory*, xvii + 449 pp.; by I. M. Gel'fand and G. E. Shilov, Academic Press, New York and London, 1977.

At the beginning of the 1950's the theory of generalized functions was in somewhat the same state that nonstandard analysis is in today. Mathematicians were by no means of one mind as regards the benefits of the theory. Critics felt that it was an overblown way of describing a modest but useful scheme for making computations in one area of harmonic analysis: the Heaviside calculus. Even those who were enthusiastic about the theory regarded distributions as shadowy entities like quarks or mirons. It was felt that to understand the theory one had first to become familiar with a formidable array of topics in abstract analysis: barrelled topological vector spaces, Montel spaces and so on. Graduate students were discouraged from going into distribution theory and advised to do Schauder-Leray estimates instead.

By the end of the 50s this situation had completely changed. Generalized functions had come to be viewed as an indispensible tool in almost every area of analysis. The reasons for this were not hard to account for. The novelty of the theory wore off and people gradually got used to thinking of distributions as house-and-garden variety objects. It turned out that the function-theoretic underpinnings of the theory could be reduced to standard facts about Sobolev spaces, so one did not need to know about espaces tonnellés. In fact to learn enough of the theory to be able to work with distributions, albeit nonrigorously, one could get by with a few elementary facts about the Fourier transform. This meant that distributions could be made a regular part of the graduate curriculum. Finally, two extremely important mathematical developments, both occurring in the middle 50s, turned out to depend on the theory of distributions in an absolutely essential way. One of these was in the area of linear partial differential equations. In 1955, Ehrenpreis, Hörmander, and Malgrange proved independently that every constant coefficient partial differential equation admits a fundamental solution. The solution is produced by making sense of $p(\xi)^{-1}$ as a generalized function when p is a polynomial function on $\mathbf{R}^n$. There are several ways of doing this, but all require Schwartz's theory of distributions.

The other development was in the area of group representations. In his thesis Bruhat was able to settle a number of fundamental questions concerning the irreducibility and unitarizability of induced representations of Lie groups by reducing them to technical questions about the kernels of intertwining operators. The mathematical tool which made this reduction possible was one of the key theorems in distribution theory, the Schwartz kernel theorem.