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Von Neumann regular rings, by K. R. Goodearl, Monographs and Studies in Mathematics, No. 4, Pitman, London-San Francisco-Melbourne, 1979, xvii + 369 pp., $46.00.

The conception of von Neumann regular rings occurred in 1936 when John von Neumann defined a *regular ring* as a ring R with the property that for each $a \in R$ there exists $b \in R$ such that $a = aba$. In order to distinguish these rings from the regular Noetherian rings of commutative algebra, noncommutative ring theorists have added von Neumann's name as a modifier. There is, however, very little chance of confusing these two concepts since their only common objects of study would be fields. The standard example of a regular ring is the complete ring of linear transformations of a vector space over a division ring.

Motivated by the coordinatization of projective geometry which was being reworked at that time in terms of lattices, von Neumann introduced regular rings as an algebraic tool for studying certain lattices. The lattices von Neumann was interested in had arisen in joint work with F. J. Murray dealing with algebras of operators on a Hilbert space [10], which subsequently came to be known as *von Neumann algebras* or *W^*-algebras*. Although a W^*-algebra A turns out to be a regular ring only when A is finite-dimensional, a regular ring can be assigned to A by working with the set $P(A)$ of projections, a projection on A being a selfadjoint idempotent. For a finite W^*-algebra A, Murray and von Neumann used a regular ring R to "coordinatize" $P(A)$ in the sense that $P(A)$ turned out to be naturally isomorphic to the lattice of principal right ideals of R. (Finite means that $tt^* = 1$ whenever $t^*t = 1$, for $t \in A$.) Expanding on this idea [14], von Neumann invented regular rings so as to coordinatize complemented modular lattices, a lattice L being coordinatized by a regular ring R if it is isomorphic to the lattice of principal right ideals of R. As von Neumann showed, almost all complemented modular lattices could be coordinatized by a regular ring.

The roots of regular rings were firmly embedded in the theory of operator algebras and lattice theory. From the purely ring-theoretic viewpoint regular rings as a subject of investigation were largely ignored for a long period of time. In N. Jacobson's bible for ring theorists [5], regular rings are mentioned only briefly (p. 210). Yet there were intimations that regular rings might be worthy of study for their own sake, since they appeared in various contexts.