

to consider situations where spacetime has a topology different from that of  $R^4$ . This classical approach is in fact rather appropriate for the subject matter.

The book's biggest drawback is its excessively formal character. Whether and how to take over a particular nonrelativistic, macroscopic idealization into relativity is only partially a question of whether the appropriate differential geometric formalism can be set up. To get a real sense of the uses and limitations of some model one also needs to analyze some specific physical situations to which the model is relevant and needs to investigate the model's relation to less phenomenological, more microscopic models. Such discussions are regrettably rare in the book. But within its own framework the book is highly competent. It will remain a useful reference for quite some time.

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*Rational quadratic forms*, by J. W. S. Cassels, London Mathematical Society Monographs No. 13, Academic Press, London-New York-San Francisco, 1978, xvi + 413 pp., \$36.50.

The focal point of the book under review, the classification of quadratic forms over  $\mathbf{Z}$ , can be formulated very simply. If

$$f = \sum f_{ij}x_i x_j \quad \text{and} \quad g = \sum g_{ij}y_i y_j$$

are nondegenerate quadratic forms in  $n$  variables  $x_1, \dots, x_n$  and  $y_1, \dots, y_n$  respectively, with coefficients  $f_{ij} = f_{ji}$  and  $g_{ij} = g_{ji}$  in  $\mathbf{Z}$ , is it possible to determine whether or not  $f$  and  $g$  are equivalent over  $\mathbf{Z}$ , i.e. whether or not there is a linear change of variables

$$y_j = \sum t_{ij}x_i$$

with  $(t_{ij})$  an invertible matrix over  $\mathbf{Z}$  which will transform  $g$  into  $f$ ? This is closely related to the question of describing those integers that are represented by  $g$ , and to the more general question of which quadratic forms are represented by  $g$  over  $\mathbf{Z}$ . All these problems can, of course, be formulated over any integral domain and not just over  $\mathbf{Z}$ . In particular, they can be formulated over an arbitrary field where it can be shown, rather simply, that every quadratic form is equivalent to a diagonal form provided the characteristic of the field is not 2. If the field in question is  $\mathbf{R}$ , then  $g$  is equivalent to a diagonal form

$$\sum_1^r x_i^2 - \sum_{r+1}^n x_j^2,$$

and  $r$  and  $n$  provide a complete set of invariants for equivalence over  $\mathbf{R}$ . This is Sylvester's Theorem. It is the classification theorem over  $\mathbf{R}$ . Forms over  $\mathbf{R}$  with  $r > 0$  and  $n > r$  are called indefinite, with  $r = n$  positive definite, and so on. Forms over  $\mathbf{Z}$  are called indefinite if they are indefinite when viewed over  $\mathbf{R}$ , and so on. It is important to make the distinction between definite and