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HOMOGENEOUS EXTENSIONS OF C^*-ALGEBRAS AND K-THEORY. I[1]

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Let L denote the bounded operators on a complex, separable, infinite-dimensional Hilbert space, K the ideal of compact operators, $Q = L/K$ the Calkin algebra, and π: $L \longrightarrow Q$ the natural map. Brown, Douglas, and Fillmore (BDF) [1], [2] initiated the study of unitary equivalence classes of extensions of C^*-algebras of the form

$$\begin{array}{ccccccccc} 0 & \longrightarrow & K & \longrightarrow & E & \longrightarrow & A & \longrightarrow & 0 \\ & & \| & & \downarrow & & \downarrow \tau & & \\ 0 & \longrightarrow & K & \longrightarrow & L & \longrightarrow & Q & \longrightarrow & 0 \end{array}$$

for fixed separable nuclear C^*-algebras A. The resulting group of equivalence classes is denoted $\mathrm{Ext}(A)$, or $\mathrm{Ext}(X)$ when $A = C(X)$, the ring of continuous complex-valued functions on a compact metric space X. In [2], BDF show that $\mathrm{Ext}(X) \cong K_1(X)$ when X is a finite complex. If X is of finite dimension then $\mathrm{Ext}(X)$ has been calculated by Kahn, Kaminker, and the author (KKS) [3]:

$$\mathrm{Ext}(X) \cong {}^sK_1(X) \overset{\text{def}}{\equiv} K^0(FX)$$

where ${}^sK_*(X) = K^*(FX)$ is Steenrod K-homology and FX is a CW-approximation for the function spectrum $\{F(X, S^n)\}$. In particular, if X is a closed subset of S^{2n} then

$$\mathrm{Ext}(X) \cong [S^{2n} - X, Q^r] \equiv K^0(S^{2n} - X)$$

where Q^r denotes the group of invertible elements of Q with the subspace topology, and $[X, Y]$ denotes basepoint-preserving homotopy classes of based maps $X \longrightarrow Y$. *Henceforth X and Y are understood to be finite-dimensional compact metric spaces.*

For a topological space Y and $*$-algebra B, the continuous functions $C(Y, B)$ form a $*$-algebra. In particular, we consider the algebra $C(Y, L_{*s})$, where L_{*s} denotes L with the strong-* topology. This is a C^*-algebra with

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