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Metric planes and metric vector spaces, by Rolf Lingenberg, John Wiley & Sons, New York, 1979, xiv + 209 pp.

Given a geometry there are canonically associated groups. For example, geometries arising from bilinear forms yield the symplectic and orthogonal groups and, in the latter case, important subgroups such as the rotation group, the spinorial kernel and the commutator subgroup. In both cases appropriate factor groups (the projective groups) produce families of simple groups. These classical groups have held a prominent position in 20th century mathematics and the algebraic aspects are treated in the classic works of Artin, Dieudonné, Eichler, and O'Meara ([A], [D1], [D2], [E], [O'M]). Of primary importance are the investigations on generators, structure, isomorphisms and automorphisms.

It is natural to pose the inverse problem–given a group G of a certain type is there a geometry associated to G and what is its character. This is part of the investigation of Tits in [T] who reminds us in the introduction of the construction of a complex projective space from $SL_n(\mathbf{C})$ by using the maximal parabolic subgroups as subspaces; incidence of subspaces occurring when the intersection of two maximal parabolics is parabolic. Along a slightly different line are the works of Bachman, Lingenberg, Sperner, and Strubecker, and the book under review.

A fundamental object and object of study in *Metric planes and metric vector spaces* is that of an *S-group*. Indeed, 5 of the 8 chapters and approximately 3/4 of the pages are devoted to this topic. This concentration will be reflected in the review.

An S group is a pair (G, S) consisting of a group G generated by a subset S of the full set J of involutions subject to the following axiom

AXIOM S

$$a \neq b \quad \textit{and} \quad abx, aby, abz \in J \quad \textit{implies} \quad xyz \in S.$$

The relation κ defined by

$$(a, b, c) \in \kappa \Leftrightarrow abc \in J \qquad (*)$$

is a *ternary equivalence relation* on the set S, meaning that κ is a subset of $S \times S \times S$ satisfying:

(E1) (*Reflexivity*). *If a, b, and c are not mutually distinct, then* $(a, b, c) \in \kappa$.

(E2) (*Symmetry*). *If* $(a, b, c) \in \kappa$ *and* π *is a permutation of* $\{a, b, c\}$ *then* $(\pi(a), \pi(b), \pi(c)) \in \kappa$.

(E3) (*Transitivity*). $a \neq b$ *and* $(a, b, c) \in \kappa$ *and* $(a, b, d) \in \kappa$ *imply that* $(a, c, d) \in \kappa$.

Axiom S is needed to verify (E3) only.

If S contains at least two elements the pair (S, κ) is called an *incidence structure*; the elements of S are called *lines* and are denoted by lower case latin letters. Three lines a, b, c are *concurrent* if and only if $(a, b, c) \in \kappa$. The subsets $S(a, b) = \{x | (a, b, x) \in \kappa\}$ are called *points* and are denoted by capital letters. Thus a point is a collection of lines all of which are concurrent