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Ordinary differential equations, by V. I. Arnold, translated from the Russian by Richard A. Silverman, MIT Press, Cambridge, Massachusetts, 1978, x + 280 pp., $8.95.

The past twenty years have witnessed a revolution in the field of ordinary differential equations. It is not uncommon to attend a seminar on differential equations and not even hear the words differential equations, let alone see one written on the board. The "in phrase" these days is dynamical systems, and the language spoken is often the language of topology and differential geometry. *Ordinary differential equations* by the famed Soviet mathematician V. I. Arnold is a superb introduction to the modern theory of differential equations, and while reviewing this book, it is instructive to take a closer look at the profound changes that have occurred in this field. We begin with the fundamental concept of a dynamical system.

Consider a system that is evolving in time. Let x denote the initial state of the system, and $g^t x$ its state at time t, with $g^0 x = x$. The set M of all possible states is called the phase space of the system, and the individual states x are called phase points. Suppose, moreover, that the mappings g^t satisfy the group property

$$g^{t+s}x = g^t(g^s x) \tag{1}$$

and that g^t and $(g^t)^{-1}$ satisfy appropriate continuity conditions. The set of mappings g^t, together with the phase space M is called a dynamical system.

Dynamical systems occur very naturally in the study of ordinary differential equations. Let

$$\dot{x} = v(x) \tag{2}$$

be a differential equation defined on a domain M of n dimensional Euclidean