BULLETIN (New Series) OF THE
AMERICAN MATHEMATICAL SOCIETY
Volume 2, Number 3, May 1980

0002-9904/80/0000-0214/$01.75

Semigroups and combinatorial applications, by Gerard Lallement, Pure and Applied Mathematics Series, Wiley, New York, 1979, xii + 376 pp., $27.50.

One way in which algebra differs from analysis is its closeness to the axioms. Whereas real numbers require a short lecture course to be properly defined, groups need only three axioms, and a theory of remarkable depth and richness springs from this modest beginning. Semigroups need even fewer axioms, and although their theory does not, and probably never will, rival group theory in importance, it has certainly seen a considerable development in recent years. If in some circles the subject has gained a reputation for dullness, this may be explained (though not justified) by the fact that it is possible to write on semigroups with little previous knowledge and as a consequence a large mass of only very loosely related facts had been accumulated. This impression was reinforced by the appearance of Clifford-Preston [1] which faithfully recorded these facts and, though excellent as a reference, was much less well suited for continuous reading. The book by Howie [3] gives a readable introduction to the subject but confines itself to what might be called 'pure' semigroup theory, leaving such important topics as codes and languages untouched. In Howie's defence it might be said that there are now excellent books on both these topics, but it would surely be desirable to have a connected treatment of all these matters between one pair of covers. Eilenberg's volumes [2] deal with both semigroups and languages in a profound and original way which will clearly have a decisive influence on the subject, but it is not (and was not intended to be) a survey of what was known. Such a survey, if really exhaustive, would probably be unreadable, but one might hope for an account stressing the interaction between semigroups and language theory. This is provided by Lallement's book which aims "to present those parts of the theory of semigroups that are directly related to automata theory, algebraic linguistics and combinatorics". Thus it fills a real gap in the current literature, and fills it rather well. The first 100 pages are pure semigroup theory, the next 200 or so deal with free semigroups, codes and languages and the rest is devoted to combinatorial questions related to (a) Burnside's problem and (b) MacMahon's Master Theorem.

The history of semigroups should be easy to write, since most of it has taken place in the last 50 years and of the references in this book well over two-thirds date back less than twenty years. Almost the first step was the theorem of Suschkewitsch (1928) describing the structure of the minimal ideal in a finite semigroup, later generalized by D. Rees (1940) to completely simple semigroups (i.e. simple semigroups with minimal left and right ideals). Let A, B be any sets, G a group and $P = (p_{\beta\alpha})$ a $B \times A$ matrix with entries in G, then the *Rees matrix semigroup* over G with *sandwich matrix* P consists of all $A \times B$ matrices with a single entry in G and the rest 0 or undefined, say $[\alpha, g, \beta]$, with multiplication $[\alpha, g, \beta][\alpha', g', \beta'] = [\alpha, gp_{\beta\alpha'}g', \beta']$. Now the Rees-Suschkewitsch theorem states that any such matrix semigroup is completely simple and conversely every completely simple semigroup is of this