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THE LANGLANDS CONJECTURE FOR Gl_2 OF A LOCAL FIELD

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Let F be a p-field and let $W(F)$ be the absolute Weil group of G. Let $A_n(F)$ be the set of (equivalence classes of) continuous semisimple n-dimensional complex representations of $W(F)$ and let $A(Gl_n(F))$ be the set of (equivalence classes of) irreducible admissible representations of $Gl_n(F)$. By local classfield theory there is a natural bijection between the sets $A_1(F)$ and $A(Gl_1(F))$, this latter set being just the set of quasi-characters of the multiplicative group $F^\times$ of F; we observe the convention of using this bijection to identify one-dimensional representations of $W(F)$ with quasi-characters of $F^\times$.

It is a conjecture of Langlands [JL] that there should exist a bijection $\sigma \longrightarrow \pi(\sigma)$ between $A_2(F)$ and the subset of nonspecial representations in $A(Gl_2(F))$, this bijection having the following properties.

1. For χ in $A_1(F)$, $\pi(\sigma \otimes \chi) = \pi(\sigma) \otimes \chi \circ \det$.

2. The one-dimensional representation $\det \sigma$ should (under our convention) be the central character of $\pi(\sigma)$.

3. $L(\sigma) = L(\pi(\sigma))$; $\epsilon(\sigma) = \epsilon(\pi(\sigma))$ where L, ϵ are the *local factors* associated to σ and $\pi(\sigma)$ [JL] with respect to some fixed character of F^+.

In case the representation σ in $A_2(F)$ is reducible or imprimitive (i.e., induced from a proper subgroup of $W(F)$) the existence of $\pi(\sigma)$ is demonstrated in [JL]; in particular, this verifies the conjecture in case $p \neq 2$.

In case $p = 2$, Yoshida [Y] and Ree [R] have shown the existence of $\pi(\sigma)$ for certain primitive representations σ and Tunnell [T] has shown that the map $\sigma \longrightarrow \pi(\sigma)$ is a bijection given that the existence of $\pi(\sigma)$ has already been established for all σ in $A_2(F)$, thus establishing the validity of the conjecture for $F = \mathbf{Q}_2$ as well as for fields F of residual characteristic two which contain the cube roots of unity.

We have recently verified the existence of $\pi(\sigma)$ for any primitive representation σ of $A_2(F)$ and we have shown that the map $\sigma \longrightarrow \pi(\sigma)$ is indeed a bijection with the properties described above. We give here a sketch of our methods; a more detailed description of our results will appear elsewhere.

1. As above, let F be a p-field, $p = 2$ and let σ be a primitive two-dimensional representation of $W(F)$. Then [W] there exists a unique extension $K = K(\sigma)$

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