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REALIZATIONS OF NONLINEAR SYSTEMS AND ABSTRACT TRANSITIVE LIE ALGEBRAS

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Introduction. We shall derive nonlinear analytic realizations in system theory from infinite transitive Lie algebras and Lie pseudogroups. By using noncommutative generating power series, this new approach allows a local input-output viewpoint which was not possible until now (cf. [5], [6], [9], [10]) and should lead, thanks to the notion of *syntactic* Lie algebra, to many developments.

I. Review of noncommutative generating power series (cf. [2], [3]). X^* is the free monoid generated by $X = \{x_0, x_1, \ldots, x_n\}$ and 1 is its identity element. Let $\mathbf{R}\langle X\rangle$ and $\mathbf{R}\langle\langle X\rangle\rangle$ be the $\mathbf{R}$-algebras of formal polynomials and power series with real coefficients and associative variables $x_j \in X$ (noncommutative if $n \geqslant 1$).

A causal functional $F(t; u_1, \ldots, u_n)$, where $u_1, \ldots, u_n$: $[0, T) \longrightarrow \mathbf{R}$ are piecewise continuous functions, is said to be *analytic* iff it is given by an element

$$g = (g, 1) + \sum_{\nu \geqslant 0} \sum_{j_0, \ldots, j_\nu = 0}^{n} (g, x_{j_\nu} \ldots x_{j_0}) x_{j_\nu} \ldots x_{j_0}$$

of $\mathbf{R}\langle\langle X\rangle\rangle$, called its *generating* power series, such that its value is

$$(1) \qquad F(t; u_i) = (g, 1) + \sum_{\nu \geqslant 0} \sum_{j_0, \ldots, j_\nu = 0}^{n} (g, x_{j_\nu} \ldots x_{j_0}) \int_0^t d\xi_{j_\nu} \ldots d\xi_{j_0}.$$

The iterated integral is defined recursively on the length

$$\xi_0(\tau) = \tau, \qquad \xi_i(\tau) = \int_0^\tau u_i(\sigma)\, d\sigma \qquad (i = 1, \ldots, n),$$

$$\int_0^\tau d\xi_j = \xi_j(\tau) \qquad (j = 0, 1, \ldots, n),$$

$$\int_0^t d\xi_{j_\nu} \ldots d\xi_{j_0} = \int_0^t d\xi_{j_\nu}(\tau) \int_0^\tau d\xi_{j_{\nu-1}} \ldots d\xi_{j_0}.$$

HYPOTHESIS (H). (1) is absolutely convergent for t and $\max_{0 \leqslant \tau \leqslant t} |u_i(\tau)|$ sufficiently small.

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