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PROJECTIONS OF C^∞ AUTOMORPHIC FORMS

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The purpose of this paper is to exhibit an explicit formula which describes the projection operator from the space of C^∞ automorphic forms to the subspace of holomorphic cusp forms, and to apply it to the zeta functions of Rankin type.

Fix a number $k > 0$ such that $2k \in Z$. Let N be a positive integer such that $N \equiv 0 \bmod(4)$ if $k \notin Z$, and let χ: $(Z/NZ) \longrightarrow \mathbf{C}$ be a Dirichlet character modulo N. Define

$$\Gamma_0(N) = \{(\begin{smallmatrix} a & b \\ c & d \end{smallmatrix}) \in SL_2(Z) | c \equiv 0 \bmod(N)\}$$

and $\mathfrak{H} = \{z = x + iy \in \mathbf{C} | y > 0\}$. For $\gamma = (\begin{smallmatrix} a & b \\ c & d \end{smallmatrix}) \in SL_2(Z)$ and $z \in \mathfrak{H}$, we put $\gamma(z) = (az + b)(cz + d)^{-1}$. For $b \geqslant 0$, denote by $\mathfrak{S}(k, N, \chi, b)$ the set of functions F satisfying

(1) F is a C^∞ function from $\mathfrak{H}$ to $\mathbf{C}$,

(2) $F(\gamma(z)) = \chi(d) j(k, \gamma, z) F(z)$ for all $\gamma \in \Gamma_0(N)$ where

$$j(k, \gamma, z) = \begin{cases} (cz + d)^k & \text{if } k \in Z, \\ \left(\frac{c}{d}\right)\left\{\left(\frac{-1}{d}\right)(cz + d)\right\}^k & \text{if } k \notin Z, \end{cases}$$

where (c/d) is the Legendre symbol (see Shimura [1] for a more complete explanation of this automorphy factor),

(3) $|F(z)| < C(y^a + y^{-b})$ for some positive real numbers C and a.

Let $G(k, N, \chi)$ be the set of all holomorphic modular forms satisfying condition (2) and let $S(k, N, \chi)$ be the subspace of $G(k, N, \chi)$ consisting of cusp forms.

Let $f \in S(k, N, \chi)$ and $F \in \mathfrak{S}(k, N, \chi, b)$. The Petersson inner product of f with F is defined as follows.

$$\langle f, F \rangle = m(N)^{-1} \int_{\Gamma_0(N) \backslash \mathfrak{H}} \overline{f(z)} F(z) y^{k-2}\, dxdy$$

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