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# RESEARCH ANNOUNCEMENTS

## ERGODIC THEORY OF AMENABLE GROUP ACTIONS. I: THE ROHLIN LEMMA

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Classically, ergodic theory began with the study of flows or actions of **R**. Later, for technical reasons, much of the theory was first developed for actions of **Z**. More recently, there has been interest in extending the theory to actions of more general groups such as $\mathbf{Z}^d$, $\mathbf{R}^d$, abelian groups etc. (see e.g. [G], [K], [KW], [L]). The natural setting seems to be amenable groups and we have been able to do much of the theory in that generality. We report here on the discrete case, so henceforth $G$ will always be countable, and focus on, what has turned out to be a very basic tool in the modern developments, the Rohlin lemma. Let $G$ denote a countable amenable group, $(X, \mathcal{B}, \mu)$ a finite nonatomic Lebesgue measure space, and $T$: $G \times X \longrightarrow X$ a measure-preserving action of $G$ on $X$. We will usually suppress $T$ and write simply $gx$ for $T(g, x)$. Recall that the action is *free* if for $\mu$ a.e. $x$, $gx \neq x$ for $g \neq 1 \in G$. Rohlin's lemma is valid for a finite set $F \subset G$, and a free action of $G$ on $(X, \mathcal{B}, \mu)$ if for all $\epsilon > 0$ there is a set $B \in \mathcal{B}$ such that

(i) $\{fB\colon f \in F\}$ are disjoint,

(ii) $\mu(FB) \equiv \mu(\bigcup_{f\in F} fB) > 1 - \epsilon$.

We say that $F$ *tiles* $G$, or that $F$ is a *tiling set* if there is a set of centers $C \subset G$ such that $\{Fc\colon c \in C\}$ is a partition of $G$.

PROPOSITION 1. *If Rohlin's lemma is valid for a finite set F for some free action of G then F tiles G.*

THEOREM 2. *If F tiles G, and G is amenable, then Rohlin's lemma is valid for F and any nonsingular free action of G.*

Thus for amenable groups, Rohlin's lemma is entirely equivalent to a purely algebraic property-tiling. In applications one wants to apply Rohlin's lemma to sets that are almost invariant in the following sense: for $K \subset G$ and $\delta > 0$, $F$

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