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Spectral theory of linear operators, by H. R. Dowson, London Math. Soc. Monographs, No. 12, Academic Press, London and New York, 1978, xii + 422 pp., $39.00.

It is a commonplace that many of the phenomena arising in pure and applied analysis can be described directly or indirectly by continuous linear operators acting on infinite-dimensional complex Banach spaces. One need only recall, for example, the Fourier-Plancherel transformation, or integral operators, or the subject of group representations. We shall call a continuous linear mapping of a complex Banach space into itself an operator (or a bounded operator). One of the most powerful tools for linking the algebraic behavior of an operator T on X with its spatial action is the *spectrum of* T, $\sigma(T)$, defined to be the set of all complex numbers λ such that $(\lambda - T)$ fails to be invertible in the algebra of all operators on X. In the general setting the spectrum plays a role analogous to that of the set of eigenvalues in the finite-dimensional case, and is a nonvoid compact set. Moreover, because the resolvent function $(\lambda - T)^{-1}$ is an analytic function of λ on the complement of $\sigma(T)$, Cauchy's integral formula can be used to define $f(T)$ whenever f is a complex-valued function analytic on a neighborhood of $\sigma(T)$. This is the "functional calculus" of T and illustrates one benefit of spectral theory–the infusion of the machinery of complex analysis into general operator theory. More broadly, spectral theory (the analysis of operators by way of their spectra) seeks to facilitate the study of operators through scalar considerations. Here are some further examples. The spectral theorem for a normal operator N on a Hilbert space H (i.e., $N^*N = NN^*$) asserts the existence of a measure $E(\cdot)$ on the Borel sets of $\sigma(N)$ having selfadjoint projections on H for its values and such that, among other things, $N = \int \lambda E(d\lambda)$. In the infinite-dimensional setting, a close analogy with finite-dimensional operators is provided by *compact operators* (operators mapping bounded sets onto sets with compact closure). Compact operators, including many integral operators, have a spectral theory which reflects their kinship with finite-dimensional operators. For instance every nonzero point in the spectrum of a compact operator is an eigenvalue with finite-dimensional eigenmanifold, and the spectrum is countable with no accumulation point except possibly the origin. Spectral theory is an extensive subject which blends with all aspects of operator theory, and the foregoing brief discussion is intended only to indicate some of its flavor.

Dowson's *Spectral theory of linear operators* enables readers with a basic understanding of Banach spaces to learn the spectral theory of bounded Banach spaces operators from the ground up, and to reach the frontiers of knowledge for three important classes of operators with a rich spectral theory (the Riesz, prespectral, and well-bounded operators). This is accomplished by careful organization of the material and an overview of the machinery of spectral theory; tne tenor of the book is decidedly toward unification and cohesion of concepts. The book is divided into five main parts, which are, in