

BRAUER GROUPS OF RATIONAL FUNCTION FIELDS

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Let $B(K)$ denote the Brauer group of a field K . The structure of $B(K)$ for K a global field (that is, either an algebraic number field or an algebraic function field in one variable over a finite constant field) was completely determined in the 1930s by the work of Albert, Brauer, Hasse, and Noether [2, Chapter 7]. In view of this result it is natural to ask next for a description of $B(K)$ when K is a function field over a global field. The purpose of this note is to announce a complete classification of the Brauer groups of rational function fields over global fields.

Before stating our main result we introduce the notation and terminology that we will use throughout this paper. p will always denote a prime and if G is an abelian torsion group, the p -primary component of G will be denoted by G_p . We denote the p -primary component of the rationals mod 1 by $Z(p^\infty)$. For G a group and α a cardinal number we denote the direct sum of α copies of G by $\bigoplus_\alpha G$. We let ω denote the cardinality of the integers and we set $T_i = \bigoplus_\omega C(2^i)$ where $C(2^i)$ is the cyclic group of order 2^i . We denote the transcendence degree over F of an extension K of F by t.d. K/F .

Let F be a global field of characteristic $q \geq 0$. For $p \neq q$, we let $\epsilon(p^t)$ denote a primitive p^t th root of unity in some algebraic closure of F . Define $\phi(F, p)$ to be the maximal r such that for p odd, $\epsilon(p^r) \in F(\epsilon(p))$, and for $p = 2$, $\epsilon(2^r) \in F(\epsilon(4))$. Let $H(F, p, n)$ denote the abelian group with generators x, y_i ($i = \phi(F, p), \phi(F, p) + 1, \dots, k, \dots$) and relations $p^{n+1}x = 0, p^i y_i = x$ ($i = \phi(F, p), \dots$). Let

$$G(F, p) = \bigoplus_\omega \left[Z(p^\infty) \oplus \bigoplus_{n=0}^\infty H(F, p, n) \right].$$

By a rational function field over F we mean a purely transcendental extension K of F with $1 \leq \text{t.d. } K/F < \infty$. With the above notation our main result is:

THEOREM. *Let K be a rational function field over the global field F of characteristic $q \geq 0$. We have:*

- (1) if $q > 0$, then $B(K)_q \cong \bigoplus_\omega Z(q^\infty)$;

Received by the editors May 15, 1979.

AMS (MOS) subject classifications (1970). Primary 16A40; Secondary 12A50.

Key words and phrases. Brauer group, class group, global field.

¹Research supported in part by NSF Grant No. MCS76-04830A01.

²Research supported in part by NSF Grant No. MCS77-01725.

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 0002-9904/79/0000-0404/\$01.75