BULLETIN (New Series) OF THE
AMERICAN MATHEMATICAL SOCIETY
Volume 1, Number 4, July 1979

ON CONJECTURES OF RIVIÈRE AND STRICHARTZ

BY CARLOS E. KENIG AND PETER A. TOMAS[1]

A. P. Calderón [1] showed that every bounded rational function of a real variable is a multiplier of L^p, $1 < p < \infty$. Littman, McCarthy and Rivière [5] showed this fails in $\mathbf{R}^n$ because $(\xi_1 - \Sigma_{j=2}^n \xi_j^2 + i)^{-1}$ is not a multiplier of $L^p(\mathbf{R}^n)$ if $1 \leqslant p < (2n + 2)/(n + 2)$. Rivière conjectured that every bounded rational function is a multiplier of $L^p(\mathbf{R}^n)$ from some $p \neq 2$. We show this is false.

THEOREM A. *Let ϕ be in $L^s \cap L^\infty(\mathbf{R})$ for some s, $0 < s < \infty$. Then, $m(\xi_1, \xi_2) = \phi(\xi_2 - \xi_1^2)$ is a multiplier of $L^p(\mathbf{R}^2)$ if and only if $p = 2$.*

R. Strichartz conjectured that there are no nontrivial Fourier multipliers of $L^p(\mathbf{R}^n)$ invariant under the action of a noncompact semisimple Lie group. For $p = 1$ this follows from the work of Greenleaf, Moskowitz and Rothchild [4]. We give a partial solution to the conjecture if $p > 1$. Let G be a noncompact connected semisimple Lie group of dimension n and rank k. If G has Lie algebra $\mathfrak{G} \simeq \mathbf{R}^n$ and Killing form B, $m(x) = \phi(B(X, X))$ is an Ad invariant function on $\mathbf{R}^n$.

We call m regular on $\mathfrak{G}$ if $\phi(t) = o(t^{-\alpha})$ for some $\alpha > 0$, when $\max(k, n - k) \leqslant 2$; no conditions are imposed if $\max(k, n - k) > 2$.

THEOREM B. *Assume ϕ is in $L^s \cap L^\infty(\mathbf{R})$ for some s, $0 < s < \infty$, and ϕ is regular on $\mathfrak{G}$. Then $m(X) = \phi(B(X, X))$ is a multiplier of $L^p(\mathbf{R}^n)$ if and only if $p = 2$.*

REMARKS. (a) Let

$$D = i\frac{\partial}{\partial X_1} + \sum_{j=2}^{n} \frac{\partial^2}{\partial X_j^2} + i.$$

Theorem A and a result of de Leeuw shows D is invertible on $L^p(\mathbf{R}^n)$ if and only if $p = 2$. E. M. Stein observed that if P is the operator with symbol $(y - x^2 + i)(y^2 + x^2 + i)$, P is invertible on some $p \neq 2$, but not all p, $1 < p < +\infty$.

Received by the editors November 16, 1978.
AMS (MOS) subject classifications (1970). Primary 42A18; Secondary 35E99.

[1]Supported in part by a grant from the NSF.

© 1979 American Mathematical Society
0002-9904/79/0000-0320/$01.75