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LINEAR ALGEBRA AND TOPOLOGY

BY SYLVAIN E. CAPPELL[1] AND JULIUS L. SHANESON[1]

Two real entried (respectively: orthogonal) $n \times n$ matrices A and B are said to be *linearly similar* if there is an invertible real (resp. orthogonal) $n \times n$ matrix C with $CAC^{-1} = B$. Of course, A, B and C may be regarded as linear functions from R^n (resp. isometries from S^{n-1}) to itself. The matrices A and B are said to be (resp. homogeneously differentiably) *differentiably similar* if there is a diffeomorphism $f\colon R^n \longrightarrow R^n$, with[2] $f(0) = 0$ (resp. $f\colon S^{n-1} \longrightarrow S^{n-1}$) for which $fAf^{-1} = B$. Differentiating this equation at 0, one sees immediately that differentiable similarity on R^n is the same as linear similarity with $C = (Df)_{(0)}$. A celebrated theorem of de Rham [7], [8] proves the corresponding equivalence for homogeneous differential similarity of orthogonal transformations of S^{n-1}.

The matrices A and B are said to be (resp. homogeneously) *topologically similar* if there is a homeomorphism $f\colon R^n \longrightarrow R^n$ with[2] $f(0) = 0$ (resp. $f\colon S^{n-1} \longrightarrow S^{n-1}$) and $fAf^{-1} = B$. For S^1, Poincaré showed, by defining rotation numbers, the equivalence of topological and linear similarity there. The study of the analogous question on S^{n-1} was continued by de Rham. In studying when topological similarity would imply linear similarity on R^n, Kuiper and Robbin [5] showed that the general problem could be reduced to the following conjecture.

TOPOLOGICAL CHARACTERIZATION CONJECTURE. *If A and B are matrices whose eigenvalues (real or complex) have modulus* 1, *then topological similarity is equivalent to linear similarity.*

This conjecture applied, for example, to all orthogonal matrices. They proved the conjecture for matrices whose eigenvalues are not roots of unity. In fact they showed that the general conjecture reduced to the case when A and B were periodic matrices, i.e. $A^n = I$, for some n. Thus, they made the conjecture:

CONJECTURE: *If A and B satisfy $A^n = I = B^n$, then topological similarity is equivalent to linear similarity.*

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[2] If the condition $f(0) = 0$ is dropped, the equivalence classes are trivially seen to be the same.