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COMPLEX ANALYSIS AND ALGEBRAIC GEOMETRY

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The theme of these four lectures is roughly "the history of, and some recent developments in, the study of algebraic geometry by analytic methods". Since this topic is much too broad I have chosen to isolate one particular analytic tool, the local notion of residue and subsequent global residue theorem, and will attempt to illustrate some ways in which residues may be used in both classical and modern problems in algebraic geometry.

The first lecture begins with the definition and basic local and global properties of the point residue in several variables. Next, both as an application of one of these local properties and for use in the third lecture, we derive some theorems of Macaulay from the classical theory of polynomial ideals. Finally we discuss the global residue theorem for the projective plane, where it will turn out to pertain to the possible configurations of points arising as the intersection of two algebraic plane curves. The simpliest special case here is the Pascal theorem, with which we conclude the lecture.

In the second talk we begin by deducing the classical form (which is in many ways more flexible than the modern version) of Abel's theorem, also from the global residue theorem for $\mathbf{P}_2$. This result is then applied to the inversion of the elliptic integral, with which much of modern algebraic geometry began and with which, at least on the number-theoretic side, it is still concerned with. Next we turn to two topics from elementary geometry. The first is the theorem of Poncelet which is given as an application of the elliptic integral, and the second is a recreational result shown to me by Joe Harris and Dave Morrison giving a geometric property of the cardioid as an application of Abel's theorem for singular curves.

Now I said previously that the motif of these lectures was to be "residues", but from here on this should probably be amended to read "residues and Hodge theory". With this in mind the second lecture represents the beginnings of our new main theme, one which will now to some extent be formalized both for curves and for higher-dimensional varieties. Following a recollection of the highlights of the relationship between an algebraic curve and its Jacobian we give a very brief sketch of some aspects of Hodge theory for general varieties. Then we turn to smooth hypersurfaces in projective space where the relation between Hodge theory and residues turns out to be quite direct. For example, using Macaulay's theorem from the first lecture it

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