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REMOVABLE SINGULARITIES IN YANG-MILLS FIELDS

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In the last several years, the study of gauge theories in quantum field theory has led to some interesting problems in nonlinear elliptic differential equations. One such problem is the local behavior of Yang-Mills fields (defined below) over Euclidean 4-space. Our main result is a local regularity theorem: A Yang-Mills field with finite energy over a 4-manifold cannot have isolated singularities. Apparent point singularities (including singularities in the bundle) can be removed by a gauge transformation. In particular, a Yang-Mills field for a bundle over R^4 which has finite energy may be extended to a smooth field over a smooth bundle over $R^4 \cup \{\infty\} = S^4$.

I would like to thank R. S. Palais for encouraging me to look at this problem.

The Yang-Mills equations are equations on the Lie algebra valued 2-form Ω, the curvature or field of a connection (or vector potential) for a bundle η over a Riemannian manifold M. For our purposes, η has structure group $G \subset SO(n)$, Lie algebra $\mathfrak{A}$, and fiber $\eta_x = R^n$. Our local estimates assume $M = U \subset R^4$, and the gauge group is the group of $\{g \in C^\infty(U, G)\}$.

A covariant derivative is a first order operator of the form

$$D = d + \Gamma, \qquad \Gamma = (\Gamma_1, \Gamma_2, \Gamma_3, \Gamma_4).$$

This transforms under an element of the gauge group to

$$g^{-1}Dg = d + g^{-1}dg + g^{-1}\Gamma g.$$

The curvature form is $\Omega = \{\Omega_{ij}\} = \{\partial/\partial x^j \Gamma_i - \partial/\partial x^j \Gamma_j + [\Gamma_i, \Gamma_j]\}$ where Γ_i and Ω_{ij} are skew $k \times k$ matrices in $\mathfrak{A}$. We say D is the covariant derivative of a Yang-Mills field Ω, if Γ is a critical point of the energy

$$E(\Gamma) = \sum_{i,j} \int - tr\, \Omega_{ij}^2\, dx = \int |\Omega|^2\, dx$$

which implies Ω satisfies the Euler-Lagrange equations

$$D^*\Omega = \sum_i \partial/\partial x^i \Omega_{ij} + [\Gamma_i, \Omega_{ij}] = 0.$$

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