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THE HOMOTOPY TYPE OF HOMEOMORPHISMS OF 3-MANIFOLDS

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The purpose of this paper is to announce some results on homeomorphism groups of 3-manifolds. These results together with the Smale conjecture (a proof of which has recently been announced by A. Hatcher) essentially reduce the computation of the homotopy type of the homeomorphism group of a 3-manifold to an analysis of a certain associated configuration space (see §2) and the homotopy types of the homeomorphism groups of the prime factors. For a prime 3-manifold, this homotopy type is known (a) if the manifold is P^2-irreducible and sufficiently large [2], (b) for $S^1 \times S^2$ and $S^1 \underset{\sim}{\times} S^2$ (see §3).

The results announced here are an extension of the first part of the first author's Ph.D. thesis and use the methods of this thesis. None of our results depends on the Smale conjecture but we have indicated where the Smale conjecture simplifies the conclusion.

1. Preliminaries. Let M be a compact 3-manifold (possibly with boundary) and let $\mathcal{H}(M)$ denote the group of PL homeomorphisms of M fixed on ∂M. This group has the same homotopy type as the (topological) homeomorphism group of M (fixed on ∂M) and also, assuming the Smale conjecture, as the diffeomorphism group of M.

Let $\mathcal{H}(M, D) \subset \mathcal{H}(M)$ denote the subgroup of homeomorphisms which are in addition fixed on a standard 3-disc D in int M. We remind you that there is a fibration:

$$\mathcal{H}(M, D) \subset \mathcal{H}(M) \longrightarrow \mathcal{E}(D, M)$$

where $\mathcal{E}(D, M)$ denotes the space of PL embeddings of D in int M. $\mathcal{E}(D, M)$ has the homotopy type

$$M \times PL_3 \quad \text{if } M \text{ is orientable,}$$

$$M \underset{\sim}{\times} PL_3 \quad \text{if } M \text{ is nonorientable,}$$

where the twisted product is defined by interchanging the components of PL_3 around orientation reversing loops in M.

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