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# BLOCKS OF CHARACTERS AND STRUCTURE OF FINITE GROUPS[1]

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**I. Introduction. Finite simple groups.** The title of my talk described a problem in which I have been interested for a very long time:

*Given a prime number p. We wish to find the relations between the properties of the p-blocks of characters of a finite group G and structural properties of G.*

Only the case is of interest that $G$ is *p-singular*, i.e., that the order $g = |G|$ is divisible by $p$.

The problem of finding all simple finite groups is still unsolved. During the last few years, very significant progress has been made by John Thompson, Daniel Gorenstein, John Walter, Helmut Bender, Michael Aschbacher, and others. It seems that most group theorists feel that it is only a matter of time until all finite simple groups will be classified. Jonathan Alperin wrote to me recently:

"It is a good guess that within five years everything should be pretty clear. But how long it will take to clean up and correct all the papers–and they do need that–is anybody's guess."

I may add that there may be some doubt about the exact meaning of a *classification* of the finite simple groups. We shall come back to this point in more detail below. While in our problem, we are concerned with arbitrary finite groups, not only simple finite groups, there cannot be any doubt about the importance of the recent developments for our problem.

Our notation will be more or less standard. By a group, we shall usually mean a finite group without mentioning this explicitly. By a *character* $\chi$ of a group $G$, we shall always mean a *complex* character. The set of irreducible characters of $G$ will be denoted by $\mathrm{Char}(G)$. The symbol $\mathrm{Cl}(G)$ will be used for the set of conjugacy classes of $G$. Since characters $\chi$ are constant on each $K \in \mathrm{Cl}(G)$, it suffices to know the value of $\chi$ for a class representative $\sigma_K \in K$. We then have the character matrix $X_G$ of $G$,

$$X_G = (\chi(\sigma_K)), \qquad (\chi \in \mathrm{Char}(G), K \in \mathrm{Cl}(G)). \tag{1.1}$$

The rows here are indexed by the $\chi \in \mathrm{Char}(G)$ and the columns are indexed by the $K \in \mathrm{Cl}(G)$. Since $\mathrm{Char}(G)$ and $\mathrm{Cl}(G)$ both consist of the same

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