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BOOK REVIEWS

On numbers and games, by J. H. Conway, Academic Press, London, New York, San Francisco, 1976, ix + 238 pp., $26.50.

Surreal numbers, by D. E. Knuth, Addison-Wesley, Reading, Massachusetts, 1974, 119 pp.,

> And more than everything my son, o beware:
> the making of many books without end;
> and excessive studiousness, tiredness of flesh.
>
> (Translated by Bill from Ecclesiastes XII, 12.)

Some readers know to play the game of nim well, fewer play a perfect annihilation game, and nobody *knows* whether there exists an opening move in chess that will *guarantee* a win for white. These games and many more, belong to the family of *combinatorial games*, by which we mean the set of all two-player perfect-information games without chance moves and with outcomes lose or win (and sometimes: dynamic tie). The motivation for ONAG may have been, and perhaps was–and I would like to think that it was–the attempt to bridge the theory gap between nim-like and chess-like games.

Why is there a gap?

Every combinatorial game can be described as a directed graph called *game-graph*, whose vertices are the game positions, and (u, v) is a directed edge if and only if there is a move from position u to position v. Denote by N the set of all positions from which the Next (first) player can force a win; by P the set of all positions from which the Previous (second) player can force a win; and by T the set of all (dynamic) Tie positions, which are positions from which no player can force a win and therefore both can avoid losing. In an acyclic game-graph there cannot be any tie positions. The N, P, T classification of any game graph $R = (V, E)$ can be determined in $O(|V| + |E|)$ steps [**8**]. For both nim and chess, a finite game-graph can be constructed and the N, P, T classification can be determined. So both games are solvable *in principle*.

If we play nim with n piles, each pile containing at most k tokens, then the game-graph contains $(k + 1)^n$ vertices. Suppose that in (generalized) chess played on an $n \times n$ board there are k different pieces. If k is about $n^2/2$, then the game-graph of chess contains $O(2^{n^2})$ vertices. So both game-graphs have exponentially many vertices, and thus both games *appear* intractable in the usual sense of computational complexity [**1**, Chapter 10], [**14**, Chapter 9], namely a computation appears to be required which is asymptotically exponential.

From a computational efficiency standpoint, the essential difference between nim and chess is that nim can be viewed as a *disjunctive compound* (sum) of independent games, namely the individual piles. A disjunctive