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G-FOLIATIONS AND THEIR CHARACTERISITC CLASSES[1]

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Introduction. Simple examples of foliations arise from submersions. Let M^n and N^q be smooth manifolds of dimensions n and q respectively, and let f: $M \to N$ be a smooth submersion, i.e. rank $(df_x) = q < n$ for all $x \in M$. Then the partition of M by the connected components of the inverse images $f^{-1}(y)$ for $y \in N$ defines a foliation of M. If the target manifold is further equipped with a G-structure in the sense of Chern [CH], where G is a closed subgroup of $GL(q)$, then the foliation of M by the components of the inverse images of the submersion f is an example of a G-foliation. Since foliations are at least locally defined by submersions as explained above, we can think of them as *relative* manifolds. In this view G-foliations are then the corresponding *relative* G-structures. This concept embraces Riemannian, conformal, symplectic, almost complex foliations, etc. In short: the classical geometry of G-structures has its relative counterpart in the geometry of G-foliations. Much progress has been made in this theory in the past half dozen years through the work of Bernstein-Rosenfeld, Bott-Haefliger, Chern-Simons, Gelfand-Fuks, Godbillon-Vey, Kamber-Tondeur, Heitsch, Thurston and many others. In this lecture we discuss selected topics in the theory of characteristic classes which are naturally attached to G-foliations. This theory is very much in flux and the present exposition is by no means a survey of even this limited field. The aim has rather been to supply a rich variety of examples together with the necessary conceptual and computational background, so as to show the attractiveness of the subject.

1. *G*-foliations and foliated bundles. For surveys on the general theory of foliations we refer to Lawson [L1], [L2]. Let M be a smooth manifold. An

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