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SOME APPLICATIONS OF THE FROBENIUS IN CHARACTERISTIC 0

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ABSTRACT. Several applications are given of the technique of proving theorems in char 0 (as well as char p) by, in some sense, "reducing" to char p and then applying the Frobenius. A "metatheorem" for reduction to char p is discussed and the proof is sketched. This result is used later to give the idea of the proof of the existence of big Cohen-Macaulay modules in the equicharacteristic case. Homological problems related to the existence of big Cohen-Macaulay modules are discussed. A different application of the same circle of ideas is the proof that rings of invariants of reductive linear algebraic groups over fields of char 0 acting on regular rings are Cohen-Macaulay. Despite the fact that this result is false in char p, the proof depends on reduction to char p. A substantial number of examples of rings of invariants is considered, and a good deal of time is spent on the question, what does it really mean for a ring to be Cohen-Macaulay?

The paper is intended for nonspecialists.

1. Introduction. The objective of this paper is to describe and relate for a general audience several areas in commutative rings and algebraic geometry in which progress has been made recently by the following general method: translate the original problem into one of showing that certain equations cannot have a solution, and then apply the Frobenius to make these equations, which at first look merely unlikely, obviously absurd. This technique, which seems a priori limited to the char $p > 0$ case, can be made to yield results for arbitrary Noetherian rings containing a field, by using the "metatheorem" (2.1) described in §2. The approximation theorem of M. Artin is the key to this kind of reduction.

Both the main results which we shall discuss in detail involve the notion of a "regular sequence" on a module. Let R be a ring (all rings are commutative, with identity) and M an R-module (i.e. a unital R-module). Then $x_1, \ldots, x_n$ in R is called a *regular sequence on M* or *M-sequence* if:

(1) $\Sigma_j x_j M \neq M$ and

(2) for each i, $1 \leqslant i \leqslant n$, x_i is not a zerodivisor on $M/\Sigma_{j<i} x_j M$.

(See [$\mathbf{AB_1}$], [$\mathbf{AB_2}$], [$\mathbf{AB_3}$], [$\mathbf{K_1}$], [**M**], [$\mathbf{N_2}$], [**Rees**], and [**ZS**].)

If R is a local ring, i.e. a Noetherian ring with a unique maximal ideal $\mathfrak{m}$, then dim R denotes, equivalently, the supremum of lengths h of chains of

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