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THE RECOGNITION PROBLEM: WHAT IS A TOPOLOGICAL MANIFOLD?

BY J. W. CANNON[1]

1. The recognition problem for topological manifolds.

1.1. DEFINITION. Let E^n denote the collection of n-tuples $x = (x_1, \ldots, x_n)$, x_i real. Define $d(x, y) = (\Sigma(x_i - y_i)^2)^{1/2}$. Then E^n becomes a metric space with metric d and is called n-dimensional Euclidean space. A (topological) n-manifold M is a separable metric space locally homeomorphic with E^n (see Figure 1).

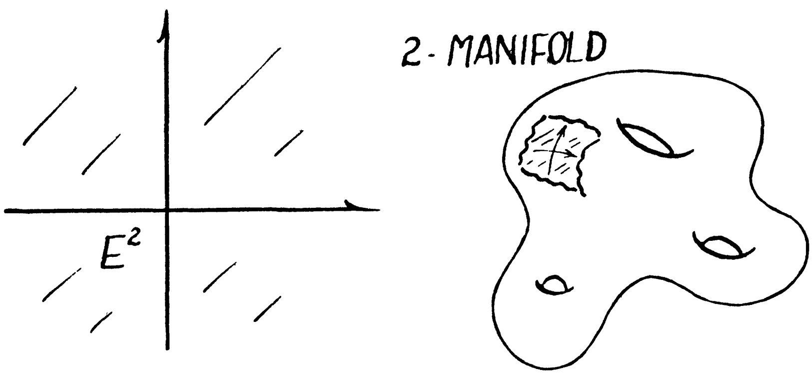

FIGURE 1

The definition of manifold, as just given, is simple. Nevertheless, it is a very difficult matter to determine whether a topological space which appears as the result of some construction in the midst of a mathematical argument is or is not a topological manifold. (See Supplement 1 for low dimensional illustrations of this difficulty.) We are thus led to the recognition problem for topological manifolds.

1.2. RECOGNITION PROBLEM. Find a short list of topological properties, reasonably easy to check, that characterize topological manifolds among topological spaces.

Recent work in geometric topology suggests that a satisfactory solution to the recognition problem for topological manifolds is imminent. The purpose of this paper is to report on that work.

A good solution to the recognition problem should make no mention of homeomorphisms as part of the hypotheses since homeomorphisms are terribly difficult to construct. A good solution probably should not involve an induction on dimension since nice submanifolds of a manifold are, in an

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