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SOME RECENT DISCOVERIES IN THE ISOMORPHIC THEORY OF BANACH SPACES[1]

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1. We shall focus on recent progress concerning one fundamental problem in Banach space theory. In so doing, we will be neglecting a vast amount of remarkable current research. In particular, we will be neglecting strong recent work on the structure of uniformly convex spaces, the location of nice finite-dimensional subspaces in general spaces, and injectivity. Before passing to our main considerations, we would just like to indicate the nature of the advances in these three other directions.

The work on uniformly convex Banach spaces is due to G. Pisier [**46**] based on earlier work of R. C. James [**30**] and P. Enflo [**15**]. It makes essential use of martingale theory to get powerful norm estimates on general uniformly convex spaces. Pisier showed that every uniformly convex space admits an equivalent norm such that all its two-dimensional spaces are "power-type" smooth and convex. Enflo had previously obtained the uniform smoothness and convexity without the power-type estimates, while James showed that nonreflexive Banach spaces always have two-dimensional spaces whose unit balls are almost square, i.e. as far from being uniformly convex or smooth as is possible. Precisely, we have the

THEOREM. *If a Banach space B is uniformly convex, then it admits an equivalent norm $\|\cdot\|$ so that there are δ, p and q with $\delta > 0$ and $1 < q \leqslant 2 < p < \infty$ such that for all x and y in B,*

$$\|x\|^p + \delta\|y\|^p \leqslant \frac{\|x+y\|^p + \|x-y\|^p}{2}$$

This is an expanded version of an invited address presented at the annual meeting of the American Mathematical Society held at Washington, D.C., on January 23, 1975; received by the editors November 10, 1977. Many of the results discussed were discovered after this meeting. In particular, much of the discussion in §3 is concerned with the solution to a problem raised at the Washington lecture.

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