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WORD PROBLEMS

BY TREVOR EVANS[1]

Introduction. In studying fundamental groups of manifolds, Dehn [4] in 1911 investigated some special cases of a problem which is now known as *the word problem for groups*. Let G be a group generated by a finite set of elements $a, b, c, \ldots$. Each element in G is a product of these generators and their inverses, for example, $a^{-1}bacb^{-1}c$. We call such expressions $w(a, b, c, \ldots)$ *words* in the generators. G is assumed to ge given by a finite set of equations or *defining relations* $r(a, b, c, \ldots) = 1$ which the generators satisfy. The question Dehn proposed was to find a uniform test or mechanical procedure (i.e. an *algorithm*) which enables us to decide whether $w = 1$ for an arbitrary word $w(a, b, c, \ldots)$ in G. We say that the word problem is *solvable* for the group G if there is such an algorithm.

A different version of the same kind of problem was posed by Thue a year or two later [35]. Consider an "abstract language" L having a finite alphabet $a, b, c, \ldots$. A word in L is a finite sequence of symbols from the alphabet, for example, *baabccba*. A language L (usually called a *Thue system*) is defined on the alphabet $a, b, c, \ldots$ by a *dictionary*, a finite set of pairs of words. If a word w is of the form usv, where u, v are words and s is a word which occurs in the dictionary paired with a word t, then we say that w can be *transformed* by the dictionary into the word utv and we write $usv \to utv$. If there is a finite sequence of transformations $w \to \cdots \to w'$ connecting two words w, w', then we say that w, w' are *equivalent* in L. Thue's problem, which we may call *the word problem for the language L*, asks for an algorithm for deciding whether two words in the language are equivalent. We say that the word problem is *solvable* for L if there is such an algorithm.

These two problems are examples of the same general situation. We have an algebra $\mathfrak{A}$ which is generated by elements $a, b, c, \ldots$. The elements of the algebra are represented by expressions (*words*) involving the generators and the operations. The algebra satisfies certain axioms and is characterized by certain basic relations $r = r'$ where the r, r' are words. We wish to find some effective test for deciding whether two words $w(a, b, c, \ldots)$, $w'(a, b, c, \ldots)$ represent the same element of the algebra, i.e. whether $w = w'$ follows from the axioms and the relations $r = r'$. The question of the existence of such an algorithm is called *the word problem* for the algebra. Obviously the word problem for groups is of this type. The word problem for the language L becomes such a question if we view L as a semigroup given by generators

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