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FIBRATIONS AND GEOMETRIC REALIZATIONS

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There is a folk theorem associated to the construction of classifying spaces for topological groups which says that a map of simplicial spaces which is a fibration in every degree has a fibration as its geometric realization. Peter May [GILS] has given a useful form of this which involves quasifibrations. This result has seemed to me a very interesting one, as it relates two rather opposite types of operations. On the one hand, it involves the geometric realization, which is defined by mapping into other things, and on the other hand, it involves fibrations, which are characterized by the properties of maps of other things into them. Any theorem which mixes "left" and "right" mapping properties should be expected to be difficult to prove. Since what is really wanted in applications is a homotopy theoretic result, the possibilities for complication are almost infinite.

It seemed to me that a proof should be found which allowed for a homotopy invariant statement of the theorem, and which had the property that the ad hoc part of the argument was isolated in a reasonably small, isolated computation. After some effort, I managed to find such a proof, which I will outline in this paper. The virtue of this proof is that it involves a number of areas of topology which are known only to experts and which deserve a wider audience. Thus the proof of the theorem about the geometric realization of fibration has become the occasion for an exposition of simplicial methods, axiomatic homotopy theory, and homotopy limits and colimits. I shall only touch on simplicial methods, as there are several texts which cover these, and spend most of my time on axiomatic homotopy theory and the theory of homotopy limits and colimits. These are subjects which I think will become more pervasive in topology as attempts are made to apply homotopy theory in ever more general settings.

For those who know what simplicial objects are, I will state the result on geometric realizations and illustrate it with some examples. For those not familiar with the terms I shall attempt to explain them as the talk proceeds.

THEOREM. *If f: $X \to Y$ is a map of simplicial spaces such that $\pi_0(f)$ is a Kan fibration, and if the higher groupoids $\Pi_\infty(X)$ and $\Pi_\infty(Y)$ are fully fibrant, then for any map g: $Y' \to Y$ of simplicial spaces, if X' is the homotopy theoretic fiber product of Y' with X over Y, $R(X')$ is the homotopy theoretic fiber product of $R(Y')$ with $R(X)$ over $R(Y)$, where R denotes the geometric realization.*

This theorem is proved for bisimplicial sets to take advantage of the rather

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