

## AN ALGEBRAIC APPROACH TO THE TOPOLOGICAL DEGREE OF A SMOOTH MAP

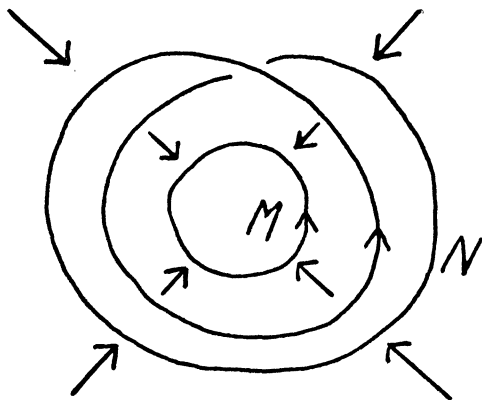
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The singularities of mappings have attracted a lot of attention lately, perhaps partly because the field touches so many others. However, many elementary problems still remain. I was attracted to the subject myself by a problem shown me by Harold Levine, which I would like to describe. Levine and I worked on this problem together, and the new results that I will discuss come from our joint work, mostly contained in [Eisenbud-Levine].

**Topological degree.** The problem concerns the computation of the degree of a continuous map

$$f: M \rightarrow N$$

between two oriented compact manifolds  $M$  and  $N$  of the same dimension  $n$ , written  $\deg f$ . One way to think of the degree of  $f$  is as the number of  $f$ -preimages of a point in the target manifold  $N$ . For example, if  $f$  is the map given below by “radial projection” from the outer circle to the inner one,



(the arrow heads on  
 $M$  and  $N$  represent  
the orientations)

then every point has two preimages, so the degree of the map is 2.

Of course, care must be taken with maps like the one in the following picture, where again  $f$  is given by “radial projection” from the outer circle to the inner:

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