introduces them in §4 in connection with the "product formula" for multiplicities. He also proves the basic "multiplicity one" theorem for $L^2(S_{\mathbf{Q}} \setminus S_{\mathbf{A}})$ for his examples. However, he does not sufficiently exploit them in this reviewer's opinion. Particularly, in his proof of the product formula he uses them only in a roundabout cumbersome way, while they could be brought to bear directly and incisively. (A technical point: in the discussion of the product formula for the hyperbolic case, class number one should be assumed, but the author does not do so explicitly.) Also the general conditions for multiplicity one are not discussed; it can fail if certain first Galois cohomology classes are nontrivial. See [Co] for an example. Also the adèlic viewpoint might have simplified and clarified §6, especially the role and structure of the oscillator or Weil representation for finite rings.

Such, in brief, is the content of the book. What of the form? The book is very carefully written (not to say proofread–there are numerous typos of a harmless sort, some amusing). The author does his best to communicate the subject as he understands it. He provides many tactical and motivational asides, so even though the material is fairly technical, the reading is usually tolerable. The author's care and his down-to-earth approach will be appreciated by those wishing to learn the subject. In the first part, he really does a very good job of presenting a lot of material with minimal prerequisites. On the negative side, one feels sometimes the forest is being lost for the trees, as for example in the failure to distinguish before §10 between true solvmanifolds and the much simpler but important subclass of nilmanifolds. It would be nice to have a redo of the same material at a much higher level of sophistication. (This is perhaps too much to ask of one author.) Also somewhat dampening is the author's very pessimistic attitude toward his subject, the more so because in several spots where the author paints in his darkest palette, as in §6 and §8, some technical improvements could make the picture rosier. However as a painstaking introduction to a subject that deserves more attention and offers considerable potential for development, and for its attractive examples, (not to mention the fact that it is the only place you can read about a considerable portion of its content) the book is a valuable contribution.

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