found and the objects detected but not yet verified. Optimal solutions which use this information cannot be determined with the methods used (see Chapter VI); policies which are "optimal" neglecting the feedback can be found, but simple adaptive policies are shown to improve on them.

(If there is a significant criticism of the work under review, it is that insufficient attention is paid to algorithms which achieve or approximate optima in relatively complex situations, as opposed to problems which admit of elegant solutions.)

I have surveyed essentially the first six chapters of the book but have not done justice to the thoroughness and clarity of the exposition nor to the numerous and helpful examples. The remaining chapters deal with approximations and with moving targets, for which some interesting results are found, though not of the same generality as the earlier work.

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Finite free resolutions, By D. G. Northcott, Cambridge Univ. Press, New York, xii + 271 pp., $29.50.

This book gives a beautifully self-contained treatment of the recent Buchsbaum-Eisenbud theory [4], [5] of finite free resolutions over a commutative ring with identity, as well as of a number of related topics (e.g. MacCrae's invariant [11]). There are two features in which the author's treatment differs from existing accounts of the subject: first, he confines himself almost entirely to elementary methods, avoiding Ext, Tor, and even exterior powers (we shall do likewise), and, second, he exploits a new notion of grade (or depth) in the non-Noetherian case which permits him to dispense entirely with the Noetherian restrictions on the ring. The very elementary form of the treatment enables the author to make accessible some fancy results from the homological theory of rings to readers with virtually no background in algebra.

Hilbert [7] gave the theory of finite free resolutions its initial impetus. Suppose that one is trying to understand a finitely generated module M over a Noetherian ring R (Noetherian means that every ideal is finitely generated, and implies that every submodule of a finitely generated module is finitely generated). To give generators $u_1, \ldots, u_{n_0}$ for M is essentially the same as to map a free module $F_0 = R^{n_0}$ onto M (the map then takes $(r_1, \ldots, r_{n_0})$ to $\Sigma_i r_i u_i$). To understand M, one simply needs to understand the kernel $\{(r_1, \ldots, r_{n_0}) \in R^{n_0}: \Sigma_i r_i u_i = 0\}$, call it $\mathrm{syz}^1 M$, of this map (of course it is not unique: it depends on the choice of generators). This kernel is called a *relation module* or *module of syzygies* for M. Note that $M \cong F_0/\mathrm{syz}^1 M$. But then, to understand $\mathrm{syz}^1 M$, it is entirely natural to choose, say, n_1 generators for $\mathrm{syz}^1 M$ (equivalently, to map $F_1 = R^{n_1}$ onto $\mathrm{syz}^1 M$) and so obtain a module of syzygies of the module of syzygies, denoted $\mathrm{syz}^2 M$. Of course, there is no reason to stop at this point, and so one can obtain a (usually infinite) sequence of modules of syzygies $\mathrm{syz}^i M$ each contained in a free module $F_{i-1} = R^{n_{i-1}}$. For each i we have a composite map $(F_i \twoheadrightarrow \mathrm{syz}^i M \hookrightarrow F_{i-1})$, call