REFERENCES

1. D. L. Burkholder, R. F. Gundy and M. L. Silverstein, *A maximal function characterization of the class H^p*, Trans. Amer. Math. Soc. **157** (1971), 137–153.

2. C. Fefferman and E. M. Stein, *H^p spaces of several variables*, Acta Math. **129** (1972), 137–193.

3. G. H. Hardy and J. E. Littlewood, *A maximal theorem with function-theoretic applications*, Acta Math. **54** (1930), 81–116.

4. Shizuo Kakutani, *Two-dimensional Brownian motion and harmonic functions*, Proc. Imp. Acad. Tokyo **20** (1944), 706–714.

5. Paul Lévy, *Processus stochastiques et mouvement Brownien*, Gauthier-Villars, Paris, 1948.

6. M. Riesz, *Über Pontezreiben mit vorgeschriebenen Anfangsgliedern*, Acta Math. **42** (1920), 145–171.

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Orderable groups, by Roberta Botto Mura and Akbar Rhemtulla, Lecture Notes in Pure and Applied Mathematics, Marcel Dekker, Inc., New York and Basel, 1977, iv + 169 pp., $19.75.

The study of ordered groups began at the end of the last century. One of the first important results was obtained by Hölder in 1901 in a paper that investigated the measurement of physical data. He used the cuts introduced by Dedekind to show that an archimedean ordered group is isomorphic to the additive group R of the real numbers. Thus the real number system is the maximal archimedean ordered group. In 1907 Hahn proved that an ordered abelian group can be embedded into a lexicographic product of copies of R. His proof necessarily starts from scratch, is about 40 pages long and is one of the more difficult proofs in mathematics. In the 50's and 60's several new proofs were derived and the theorem was extended to partially ordered abelian groups and even to partially ordered sets.

Hahn realized that his lexicographic products were ordered fields provided that the index set is an ordered group. In fact, most of the early papers on ordered groups are related to the theory of ordered fields. This led to the beautiful Artin-Schreier theory of real closed fields (1926) and to the solution of Hilbert's 17th problem. Later Mal′cev (1948) recognized the connection between ordered groups and the embedding of integral domains into division rings and cancellative semigroups into groups, Neumann and others constructed ordered division rings by extending Hahn's ideas to nonabelian groups. In particular Mal′cev (1948) and Neumann (1949) showed that the group ring of an ordered group over an ordered division ring can be embedded in an ordered division ring, Hilbert in his Grundlagen der Geometrie showed that each ordered group can be embedded in an ordered division ring.

In the 30's and 40's the theory of ordered abelian groups branched out into two areas: (I) partially ordered abelian groups and rings, and (II) ordered nonabelian groups, which is the subject of these notes. In 1935 Kantorovich started his investigation of partially ordered linear spaces, which was continued through the war years by Kantorovich and his pupils. Also during