British mathematical books have wretched indices. This one maintains the tradition.

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Brownian motion, Hardy spaces and bounded mean oscillation, by K. E. Petersen, London Mathematical Society Lecture Note Series (2) vol. 28, Cambridge University Press, Cambridge, London, New York, Melbourne, 105 pp.

In recent years the techniques and theorems of Brownian motion have been used to prove theorems about harmonic and analytic functions. It is always pleasant when two branches of mathematics which ostensibly have little to do with one another can help each other out. There are two main links which allow Brownian motion (roughly representing the paths of an idealized random traveller) to be connected to the theory of harmonic and analytic functions. Kakutani [4] showed that Brownian motion can be used to solve the Dirichlet problem. Dispensing with the technicalities of continuity, smoothness, and measurability, here is what Kakutani's theorem says: Let S be an open set in $\mathbf{R}^n$ and let u be a real-valued function defined on ∂S. Let $z \in S$ and consider a typical Brownian path γ_z starting at z. Let $s(\gamma_z)$ denote the point of ∂S at which γ_z first hits ∂S. Define $\hat{u}(z)$ to be the average value of $u(s(\gamma_z))$, where the average is taken over all Brownian paths γ_z. Then $\hat{u}$ is a harmonic function on S with boundary values u.

A theorem of Lévy [5] links Brownian motion to analytic functions defined in the plane. This thoerem states that a nonconstant analytic function composed with Brownian motion is also Brownian motion, although the time scale must be changed on each Brownian path. The inituition behind Lévy's result is that an analytic function preserves angles, so that the randomness of direction is preserved. Since an analytic function need not preserve lengths, an adjustment of the time scale is necessary.

For $0 < p < \infty$ and u a function defined on the open unit disk D of the complex plane, define